DISEÑO DE EXPERIMENTOS (UN FACTOR)

Modelo: $Y_i = \mu_i + U_i = \mu + \alpha_i + U_i \qquad U_i \sim N(0;\sigma) \;\; i = 1, ..., I$

Muestra:

$Y_{ij} \sim N(\mu_i;\sigma^2)$ independientes; $\; i = 1, ..., I\,;\;\; j = 1., ..., n_i\,;\;\; \sum_i n_i = n.$

$$\hat{\mu}_i = \bar{y}_{i\cdot} = \frac{1}{n_i}\sum_j y_{ij}\;; \qquad \hat{\mu} = \bar{y}_{\cdot\cdot} = \frac{1}{n}\sum_i n_i \bar{y}_{i\cdot}\;; \qquad \hat{\sigma}^2 = S_R^2 = \frac{1}{n-I}\sum_i \sum_j (y_{ij} - \bar{y}_{i\cdot})^2$$

$$IC_{1-\alpha}(\mu_i) = \left(\bar{y}_{i\cdot} \pm t_{n-I;\alpha/2}\; S_R\sqrt{\frac{1}{n_i}}\right) \qquad IC_{1-\alpha}(\sigma^2) = \left(\frac{(n-I)S_R^2}{\chi^2_{n-I;\alpha/2}}\;;\;\frac{(n-I)S_R^2}{\chi^2_{n-I;1-\alpha/2}}\right)$$

Tabla ANOVA

Suma de cuadrados	g.l.	Varianza	Estadístico
$SCE = \sum_i n_i(\bar{y}_{i\cdot} - \bar{y}_{\cdot\cdot})^2$	$I-1$	$\frac{SCE}{I-1}$	$F = \frac{SCE/(I-1)}{SCR/(n-I)}$
$SCR = \sum_i \sum_j (y_{ij} - \bar{y}_{i\cdot})^2$	$n-I$	$S_R^2 = \frac{SCR}{n-I}$	
$SCT = \sum_i \sum_j (y_{ij} - \bar{y}_{\cdot\cdot})^2$	$n-1$		

$$IC_{1-\alpha}(\mu_i - \mu_j) = \left(\bar{y}_{i.} - \bar{y}_{j.} \pm t_{n-I;\alpha/2}\; S_R\sqrt{\frac{1}{n_i} + \frac{1}{n_j}}\right) \qquad ; \qquad S_R^2 = \frac{\sum_i (n_i - 1)s_i^2}{n-I}$$

DISEÑO DE EXPERIMENTOS (DOS FACTORES SIN INTERACCIÓN)

Modelo y muestra:

$Y_{ij} = \mu + \alpha_i + \beta_j + U_{ij} \qquad U_{ij} \sim N(0;\sigma)$ independientes; $i = 1,\ldots,I$; $j = 1,\ldots,J$

$$
\begin{aligned}
\hat{\mu} &= \bar{y}_{..} = \frac{1}{IJ}\sum_i\sum_j y_{ij} \\
\hat{\alpha}_i &= \bar{y}_{i.} - \bar{y}_{..} = \frac{1}{J}\sum_j y_{ij} - \bar{y}_{..} \\
\hat{\beta}_j &= \bar{y}_{.j} - \bar{y}_{..} = \frac{1}{I}\sum_i y_{ij} - \bar{y}_{..} \\
\hat{\sigma}^2 &= S_R^2 = \frac{1}{(I-1)(J-1)}\sum_i\sum_j (y_{ij} - \bar{y}_{i.} - \bar{y}_{.j} + \bar{y}_{..})^2 = \frac{1}{(I-1)(J-1)}\sum_i\sum_j \hat{e}_{ij}^2
\end{aligned}
$$

Tabla ANOVA

Suma de cuadrados	g.l.	Varianza	Estadístico
$SCE(\alpha) = J\sum_i \hat{\alpha}_i^2$	$I-1$	$\frac{SCE(\alpha)}{I-1}$	$F(\alpha)$
$SCE(\beta) = I\sum_j \hat{\beta}_j^2$	$J-1$	$\frac{SCE(\beta)}{J-1}$	$F(\beta)$
$SCR = \sum_i\sum_j \hat{e}_{ij}^2$	$(I-1)(J-1)$	$\frac{SCR}{(I-1)(J-1)}$	
$SCT = \sum_i\sum_j (y_{ij} - \bar{y}_{..})^2$	$IJ-1$		

Estadísticos F: $\quad F(\alpha) = \dfrac{SCE(\alpha)/(I-1)}{SCR/(I-1)(J-1)}$; $\quad F(\beta) = \dfrac{SCE(\beta)/(J-1)}{SCR/(I-1)(J-1)}$

$$IC_{1-\alpha}(\alpha_i - \alpha_j) = \left(\bar{y}_{i.} - \bar{y}_{j.} \pm t_{(I-1)(J-1);\alpha/2}\; S_R\sqrt{\frac{1}{J} + \frac{1}{J}}\right)$$

$$IC_{1-\alpha}(\beta_i - \beta_j) = \left(\bar{y}_{.i} - \bar{y}_{.j} \pm t_{(I-1)(J-1);\alpha/2}\; S_R\sqrt{\frac{1}{I} + \frac{1}{I}}\right)$$

DISEÑO DE EXPERIMENTOS (DOS FACTORES CON INTERACCIÓN)

Modelo:

$Y_{ij} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + U_{ij}$; $U_{ij} \sim N(0;\sigma)$ independientes; $i = 1, ..., I$; $j = 1., ..., J$

Muestra:

$Y_{ijk} \sim N(\mu + \alpha_i + \beta_j + (\alpha\beta)_{ij}; \sigma^2)$ independientes; $i = 1, ..., I$; $j = 1., ..., J$; $k = 1, ..., K$

$$\begin{aligned}
\hat{\mu} &= \bar{y}_{...} = \frac{1}{IJK}\sum_i\sum_j\sum_k y_{ijk} \\
\hat{\alpha}_i &= \bar{y}_{i..} - \bar{y}_{...} = \frac{1}{JK}\sum_j\sum_k y_{ijk} - \bar{y}_{...} \\
\hat{\beta}_j &= \bar{y}_{.j.} - \bar{y}_{...} = \frac{1}{IK}\sum_i\sum_k y_{ijk} - \bar{y}_{...} \\
(\hat{\alpha\beta})_{ij} &= \bar{y}_{ij.} - \bar{y}_{i..} - \bar{y}_{.j.} + \bar{y}_{...} = \frac{1}{K}\sum_k y_{ijk} - \bar{y}_{i..} - \bar{y}_{.j.} + \bar{y}_{...} \\
\hat{\sigma}^2 &= S_R^2 = \frac{1}{IJ(K-1)}\sum_i\sum_j\sum_k (y_{ijk} - \bar{y}_{ij.})^2 = \frac{1}{IJ(K-1)}\sum_i\sum_j\sum_k \hat{e}_{ij}^2
\end{aligned}$$

Tabla ANOVA

Suma de cuadrados	G.l.	Varianza	F
$SCE(\alpha) = JK\sum_i \hat{\alpha}_i^2$	$I-1$	$\frac{SCE(\alpha)}{I-1}$	$F(\alpha)$
$SCE(\beta) = IK\sum_j \hat{\beta}_j^2$	$J-1$	$\frac{SCE(\beta)}{J-1}$	$F(\beta)$
$SCE(\alpha\beta) = K\sum_i\sum_j (\hat{\alpha\beta})_{ij}^2$	$(I-1)(J-1)$	$\frac{SCE(\alpha\beta)}{(I-1)(J-1)}$	$F(\alpha\beta)$
$SCR = \sum_i\sum_j\sum_k \hat{e}_{ij}^2$	$IJ(K-1)$	$\frac{SCR}{IJ(K-1)}$	
$SCT = \sum_i\sum_j\sum_k (y_{ijk} - \bar{y}_{...})^2$	$IJK-1$		

Estadísticos F:

$$F(\alpha) = \frac{SCE(\alpha)/(I-1)}{SCR/IJ(K-1)}\ ; \quad F(\beta) = \frac{SCE(\beta)/(J-1)}{SCR/IJ(K-1)}\ ; \quad F(\alpha\beta) = \frac{SCE(\alpha\beta)/(I-1)(J-1)}{SCR/IJ(K-1)}$$

$$IC_{1-\alpha}(\alpha_i - \alpha_j) = \left(\bar{y}_{i.} - \bar{y}_{j.} \pm t_{IJ(K-1);\alpha/2}\ S_R\sqrt{\frac{1}{JK} + \frac{1}{JK}}\right)$$

$$IC_{1-\alpha}(\beta_i - \beta_j) = \left(\bar{y}_{.i} - \bar{y}_{.j} \pm t_{IJ(K-1);\alpha/2}\ S_R\sqrt{\frac{1}{IK} + \frac{1}{IK}}\right)$$

REGRESIÓN LINEAL SIMPLE

Modelo: $Y_x \sim N(\beta_0 + \beta_1 x; \sigma^2)$

$$\begin{aligned}
\hat{\beta}_1 &= \frac{cov}{v_x} \\
\hat{\beta}_0 &= \bar{y} - \frac{cov}{v_x}\bar{x} = \bar{y} - \hat{\beta}_1\bar{x} \\
\hat{\sigma}^2 &= S_R^2 = \frac{1}{n-2}\sum_i (y_i - \hat{y}_i)^2 = \frac{1}{n-2}\sum_i (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)^2
\end{aligned}$$

$$\begin{aligned}
IC_{1-\alpha}(\beta_0) &= \left(\hat{\beta}_0 \pm t_{n-2;\alpha/2}\ S_R\sqrt{\frac{1}{n} + \frac{\bar{x}^2}{nv_x}}\right) \\
IC_{1-\alpha}(\beta_1) &= \left(\hat{\beta}_1 \pm t_{n-2;\alpha/2}\ S_R\sqrt{\frac{1}{nv_x}}\right) \\
IC_{1-\alpha}(\sigma^2) &= \left(\frac{(n-2)S_R^2}{\chi^2_{n-2;\alpha/2}}\ ;\ \frac{(n-2)S_R^2}{\chi^2_{n-2;1-\alpha/2}}\right)
\end{aligned}$$

Tabla ANOVA

Suma de cuadrados	G.l.	Varianza	Estadístico
$SCE = \sum_i (\hat{y}_i - \bar{y})^2$	1	$\frac{SCE}{1}$	$F = \frac{SCE/1}{SCR/(n-2)}$
$SCR = \sum_i (y_i - \hat{y}_i)^2$	$n-2$	$\frac{SCR}{n-2}$	
$SCT = \sum_i (y_i - \bar{y})^2$	$n-1$		

$SCT = nv_y$; $SCR = nv_y(1-r^2)$; donde $r = \frac{cov}{\sqrt{v_x v_y}}$

$$IC_{1-\alpha}(\text{valor medio de } Y) = \left(\hat{y}_0 \pm t_{n-2;\alpha/2}\ S_R\sqrt{\frac{1}{n} + \frac{(x_0-\bar{x})^2}{nv_x}}\right)$$

$$IC_{1-\alpha}(\text{valor individual de } Y) = \left(\hat{y}_0 \pm t_{n-2;\alpha/2}\ S_R\sqrt{1 + \frac{1}{n} + \frac{(x_0-\bar{x})^2}{nv_x}}\right)$$

REGRESIÓN LINEAL MÚLTIPLE

Modelo: $Y_{(x_1,\dots x_k)} \sim N(\beta_0 + \beta_1 x_1 + ... + \beta_k x_k; \sigma^2)$

$$\hat{\sigma}^2 = S_R^2 = \frac{1}{n-k-1}\sum_i (y_i - \hat{y}_i)^2 = \frac{1}{n-k-1}\sum_i (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_{1i} - \cdots - \hat{\beta}_k x_{ki})^2$$

$$IC_{1-\alpha}(\beta_i) = \left(\hat{\beta}_i \pm t_{n-k-1;\alpha/2}(\text{error típico de } \hat{\beta}_i)\right),\ i = 0, \ldots, k$$

$$H_0 : \beta_i \leq 0 \quad \Rightarrow \quad \mathcal{R} = \left\{ \frac{\hat{\beta}_i}{\text{error típico de } \hat{\beta}_i} > t_{n-k-1;\alpha} \right\}$$

$$H_0 : \beta_i \geq 0 \quad \Rightarrow \quad \mathcal{R} = \left\{ \frac{\hat{\beta}_i}{\text{error típico de } \hat{\beta}_i} < t_{n-k-1;1-\alpha} \right\}$$

Tabla ANOVA

Suma de cuadrados	G.l.	Varianza	Estadístico
$SCE = \sum_i (\hat{y}_i - \bar{y})^2$	k	$\frac{SCE}{k}$	$F = \frac{SCE/k}{SCR/(n-k-1)}$
$SCR = \sum_i (y_i - \hat{y}_i)^2$	$n-k-1$	$\frac{SCR}{n-k-1}$	
$SCT = \sum_i (y_i - \bar{y})^2$	$n-1$		

$$R^2 = \frac{SCE}{SCT} = \frac{SCT - SCR}{SCT}\ ; \qquad F = \frac{R^2}{1-R^2}\,\frac{n-k-1}{k}$$

REGRESIÓN LOGÍSTICA

$$\text{Prob}(X_i = 1) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x_{1i} + \cdots + \beta_k x_{ki})}} \qquad \text{para} \quad i = 1, 2, \ldots, n$$

$$\text{IC}_{1-\alpha}(\beta_j) = \left(\hat{\beta}_j \pm z_{\alpha/2} \cdot (\text{error típico de } \hat{\beta}_j)\right) \qquad \text{para} \quad i = 0, 1, 2, \ldots, n$$

Regla de clasificación de los individuos:

- Si $\hat{\beta}_0 + \hat{\beta}_1 x_1 + \cdots + \hat{\beta}_k x_k > 0$, clasificamos $Y = 1$
- Si $\hat{\beta}_0 + \hat{\beta}_1 x_1 + \cdots + \hat{\beta}_k x_k < 0$, clasificamos $Y = 0$
