Errata to: "Transfer Methods for o-minimal Topology" [JSL 68 (2003) 785-794]

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The definition of "definable orientation" in section 5 of [1] is not correct. Where *it says*: "for each point there is a definably compact neighbourhood (of the point) N and a class...". *It should say*: "for each proper m-ball N of Y there is a class...". (See section 4 in [1] for the definition of proper m-ball.) So the correct definition is:

Definition. A definable orientation of a definable manifold Y of dimension m is a map s which assigns to each point $y \in Y$ a generator $s(y)$ of the local definable homology group $H_m^{def}(Y, Y-y)$ and which is locally constant in the following sense: for each proper m-ball B of Y there is a class $\zeta_B \in H_m^{def}(Y, Y-B)$ such that for each $p \in B$ the natural homomorphism $j_p^N : H_m^{def}(Y, Y-B) \to H_m^{def}(Y, Y-p)$, induced by the inclusion map $(Y, Y-B) \to (Y, Y-p)$, sends ζ_B into $s(p)$.
Remark. j_p^B is actually an isomorphism.

With this new definition the proof of Theorem 5.2 in [1] (the existence and unicity of a generator of $H_m^{def}(X)$ compatible with a given orientation) should be changed accordingly as follows. As in [1], we prove the stronger result:
Theorem. *If N is a definably compact subset of a definable manifold Y of dimension m with a definable orientation s, then there is one and only one class $\zeta_N \in H_m^{def}(Y, Y-N)$ such that for each $p \in N$, J_p^N maps ζ_N to $s(p)$.*
Proof. First observe that the proof of this statement, as it is in [1], proves the unicity of the relative homology class ζ_N. To prove the existence we use the unicity and we have to consider the following cases:

Case (a). N is contained in a proper m-ball of Y. Then the existence of ζ_N is ensured by definition.

Case (b). $N = N_1 \cup N_2$ and there exist ζ_{N_1} and ζ_{N_2} both satisfying the above result. Then using a suitable Mayer-Vietoris sequence (as in case 2 of [1]) we can ensure the existence of the required ζ_N.

Case (c): N is an arbitrary definably compact subset of Y. Then we argue as in case 5 of [1] to get first finitely many definably compact subsets $N_1, \dots, N_k$ of Y such that $N = N_1 \cup \dots \cup N_k$ and each N_i is contained in a proper m-ball of Y, and then the result is obtained by induction on k using cases (a) and (b). □

Note that an m-dimensional definable group G, equipped with its definable manifold structure, has a map s defined as in [1] (choose a generator $s(x) \in H_m^{def}(G, G-x)$ at a point x of a given definably connected component of G and extend s to the whole component by left group multiplication). A routine verification shows that such a map s is a definable orientation according to the new definition (so the proof of Corollary 3.4 in [1] remains the same). To see this one uses the remark above and the fact that the composition $(G, G-B) \xrightarrow{i} (G, G-x) \xrightarrow{l_{yx^{-1}}} (G, G-y)$ is definably homotopic to $(G, G-B) \xrightarrow{j} (G, G-y)$, where i and j are inclusions and l_z is left multiplication by z.

Finally note that with the incorrect definition of orientation given in [1] leads to various pathologies such as the following. Let X be the unit circle in an o-minimal non-archimedean expansion of a field. Fix a point $x_0 \in X$ in the circle and let $I \subset X$ be the infinitesimal neighbourhood of $x_0 \in X$ (a non-definable set). Then according to the definition of orientation given in [1] we could orient I in the clockwise direction and the complement of I in X in the opposite direction. Clearly it cannot exist a generator of $H_1^{def}(X)$ compatible with this "orientation" of X.

References

[1] A.Berarducci and M.Otero, Transfer methods for o-minimal topology, J.Symbolic Logic 68 (2003) 785-794.

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