Due on Wednesday 12th. Just some computations, and some reminders.

1) Let $\{B_t\}_{t\geq 0}$ be a standard Brownian motion. Compute $E(B_t^4)$. Suggestions: either make a change of variable and use the Gamma function, or utilize the moment generating function.

2) Let $X_t := B_t^2$. Find the expected value $m_X(t)$ and the covariance function $\Gamma_X(r,s)$ of the process $\{X_t\}_{t\geq 0}$.

3) Sea B_t un movimiento browniano estándar, sea $\mathcal{A}_n := \sigma(B_0, B_1, \ldots, B_n)$ y sea $X_n := B_n$.
a) Decidir razonadamente si $X := \{X_n\}_{n=0}^{\infty}$ es una martingala con respecto a $\{\mathcal{A}_n\}_{n=0}^{\infty}$.
b) Decidir razonadamente si $X = \{X_n\}_{n=0}^{\infty}$ está en L^2.
c) Decidir razonadamente si existe una $X_\infty \in L^1$ tal que $\lim_n \|X_\infty - X_n\|_1 = 0$.

4) Let $\Omega := \{0,1\}$ with the uniform probability, and let $Y : \Omega \to \Omega$ be the identity, so Y is a Bernoulli trial. Let $X_t := tY$, and let $\{\mathcal{A}_t\}_{t\in[0,\infty)}$ be the natural filtration of the process $\{X_t\}_{t\in[0,\infty)}$. Determine whether $\{\mathcal{A}_t\}_{t\in[0,\infty)}$ is right continuous.

5) Let $\{\mathcal{A}_t\}_{t\geq 0}$ be a right continuous filtration. Determine whether the notion of stopping time obtained if one uses $\{T < t\} \in \mathcal{A}_t$ as the definition, instead of $\{T \leq t\} \in \mathcal{A}_t$, coincides with the usual notion.

6) Let $\{B_t\}_{t\geq 0}$ be a standard Brownian motion. Show that the following are martingales with respect to any admissible filtration: a) $\{B_t\}_{t\geq 0}$, b) $\{B_t^2 - t\}_{t\geq 0}$, and c) for any real number u, $\{e^{uB_t - \frac{u^2 t}{2}}\}_{t\geq 0}$.