Due on Monday 29th.

1) Probar que si μ es una medida en el espacio X, y $f : X \to [0, \infty)$ es medible, entonces para todo $1 \leq p < \infty$, $\int_X f^p d\mu = \int_0^\infty pt^{p-1}\mu\{f \geq t\}dt$.

2) Prove: if $X = \{X_t\}_{t\in T}$ is uniformly integrable, then X is bounded in L^1.

3) Prove: if $X = \{X_t\}_{t\in T}$ is bounded in L^p for some $p > 1$, then X is uniformly integrable.

4) Prove or disprove: if $X = \{X_n\}_{n\geq 0}$ is a martingale bounded in L^∞ and X_∞ is its pointwise almost sure limit, then $\lim_n \|X_\infty - X_n\|_\infty = 0$.

5) The mean and any median of a r.v. cannot be very far apart. Prove that their distance is bounded by one standard deviation. HHH: use Jensen, use a minimisation property, use Jensen.