Se asume siempre que estamos trabajando en un espacio de probabilidad $(\Omega, \mathcal{A}, P)$. We also assume that the notation makes sense, so for instance, if we are talking about a martingale, there is a filtration, etc..

Due on Monday 15/10/18.

1) Let $\{X_n\}_{n\geq 0}$ and W be random variables. Suppose $W : \Omega \to \mathbb{N} \cup \{\infty\}$ and $S_W := \sum_{i=0}^{W} X_i \in L^1$. Determine whether or not the random sum S_W satisfies $E(S_W|W) = \sum_{i=0}^{W} E(X_i|W)$. Justify your answer.

There are several versions of Wald's identity on random sums. Prove the following two:

2) Let $\{X_n\}_{n\geq 1}$ be a sequence of i.i.d.r.v. in L^1 and let $W : \Omega \to \mathbb{N} \cup \{\infty\}$ be integrable and independent of the X_i's. Then $E(S_W) = E(W)E(X_1)$.

3) Here is a martingale variant: Prove that if $\{X_n\}_{n\geq 1}$ is a sequence of i.i.d.r.v. in L^1, $\mathcal{A}_n := \sigma(X_1, \ldots, X_n)$ and T is an integrable stopping time with respect to $\{\mathcal{A}_n\}_{n\geq 1}$, then the random sum $S_T(w) := \sum_{n=1}^{T(w)} X_n(w)$ satisfies $E(S_T) = E(T)E(X_1)$.