

Due on Wednesday 29th.

- 1) Let $\{N_t\}_{t \geq 0}$ be a Poisson process with rate λ . Determine whether $\lim_{t \rightarrow \infty} \text{Var}(t^{-1}N_t)$ exists, and if so, find it.
- 2) Let $\Omega := \{0, 1\}$ with the uniform probability, and let $Y : \Omega \rightarrow \Omega$ be the identity, so Y is a Bernoulli trial. Let $X_t := tY$, and let $\{\mathcal{A}_t\}_{t \in [0, \infty)}$ be the natural filtration of the process $\{X_t\}_{t \in [0, \infty)}$. Determine whether $\{\mathcal{A}_t\}_{t \in [0, \infty)}$ is right continuous.
- 3) Let $\{\mathcal{A}_t\}_{t \geq 0}$ be a right continuous filtration. Determine whether the notion of stopping time obtained if one uses $\{T < t\} \in \mathcal{A}_t$ as the definition, instead of $\{T \leq t\} \in \mathcal{A}_t$, coincides with the usual notion.