

Due on Monday 20th.

- 1) Let $\Omega := \{0, 1\}$ with the uniform probability, and let $Y : \Omega \rightarrow \Omega$ be the identity, so Y is a Bernoulli trial. Let $X_t := tY$, and let $\{\mathcal{A}_t\}_{t \in [0, \infty)}$ be the natural filtration of the process $\{X_t\}_{t \in [0, \infty)}$. Determine whether $\mathcal{A}_0 = \mathcal{A}_0^+$.
- 2) In a Galton-Watson branching process, let X_n denote the number of individuals belonging to the n -th generation, and let μ denote the expected number of offsprings of any given individual. Determine whether the process $\{X_n/\mu^n\}_{n \geq 0}$ is a martingale with respect to the natural filtration.
- 3) The mean and any median of a r.v. cannot be very far apart. Prove that their distance is bounded by one standard deviation. HHH: use Jensen, use a minimisation property, use Jensen.