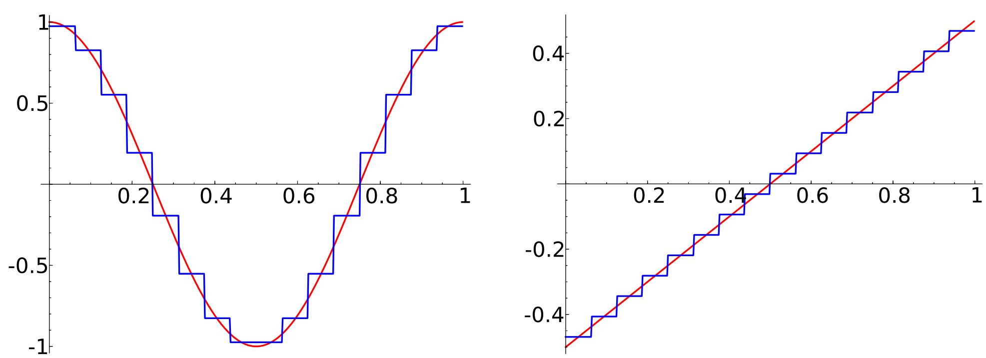
Experimental challenge*: **Step by step**

Wavelets: theory and practice — Deadline: End of the course

Experimental part

Choose a 1-periodic signal $f = f(t)$ with zero average on each period and such that you know an explicit formula for $F(t) = \int_0^t f$. This is the case, for instance, for a cosine wave or a sawtooth signal.

Write code with the software you prefer to find the contribution of $j < J$ to $\sum\sum c_{jk}\psi_{jk}$, the wavelet expansion of $f\chi_{[0,1]}$ with ψ the Haar wavelet. Plot the result for some values of J.



Cosine signal for J = 4 — Sawtooth for J = 4

If you sample the result and repeat the data a number of times you can hear an approximation of the original signal quantized in time. As we saw in the course (I posted an audio file on the web), for a pure tone sometimes a strange high pitch appears. This is due to the high frequencies introduced by the sharp jumps.

Mathematical part

Prove that this procedure of truncating the wavelet expansion to $j < J$ is actually equivalent to quantize in time substituting the signal by the step function that gives its average on each interval $\big[2^{-J}k, 2^{-J}(k+1)\big)$, $k \in \mathbb{Z}$. This is very simple in many ways, for instance thinking about the multiresolution analysis.

*Some experiments are classical, some are my idea and others come from specific sources. In the latter case I have omitted the reference here on purpose to force the students to work on their own. If you are the author, please do not get angry. I intend to incorporate the references to the final version of the notes.