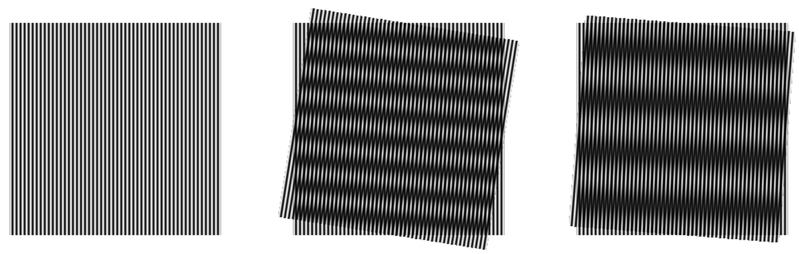
Experimental challenge*: **Moiré, qu'est-ce que c'est?**

Introduction to digital signals — Deadline: 18/April/2018

Experimental part

Print very close black dots, straight lines or curves on a couple of transparent slides and play to overlap them and observe some funny patterns, it is the so called, *moiré effect.* You can also use gray tones. To my taste one of the best results is achieved with thin circular sectors (see the links in the web page of the course) but it appears even in simple situations.



Single slide — angle $\pi/20$ — angle $\pi/40$

Try to reproduce at least the experiment above with vertical lines. For large angles nothing peculiar happens, but for small angles the pattern appears.

Mathematical part

The grid depicted above actually corresponds to the gray tone $C(t) = \big(F(t) - m\big)/(M - m)$ with $F(t) = \sum_{n\in\mathbb{Z}} e^{-\pi(n-\nu t)^2}$ and m and M are the minimum and the maximum of F to normalize it between 0 (black) and 1 (transparent). Explain why the natural model for any color function C depending on one variable is that superposition with angle α corresponds to the color function $G(x, y) = C(x)C(x\cos\alpha + y\sin\alpha)$.

The (2D) Fourier transform formally detects periodicity through Dirac deltas. On the other hand, at a certain distance we cannot distinguish frequencies beyond a certain threshold V. So if you compute $\widehat{G}$ and look for values $\|\vec{\xi}\| \le V$ such that $\widehat{G}(\vec{\xi})$ has the highest peaks (coefficients of the Dirac deltas) then $\vec{\xi}$ indicates the direction in which the pattern repeats and $\|\vec{\xi}\|$ its frequency.

Explain the figures using $F(t) = \sum_{n\in\mathbb{Z}} e^{-\pi n^2} e(\nu n t)$ and this model with V much smaller than ν. Prove that using grids of different frequencies ν_1 and ν_2 the frequency of the pattern is $\sqrt{\nu_1^2 + \nu_2^2 - 2\nu_1\nu_2\cos\alpha}$. It is interesting because it allows to measure tiny distances observing macroscopic effects. You can find much simpler geometric explanations for this example of vertical lines but Fourier analysis applies to more complicate cases.

*Some experiments are classical, some are my idea and others come from specific sources. In the latter case I have omitted the reference here on purpose to force the students to work on their own. If you are the author, please do not get angry. I intend to incorporate the references to the final version of the notes.