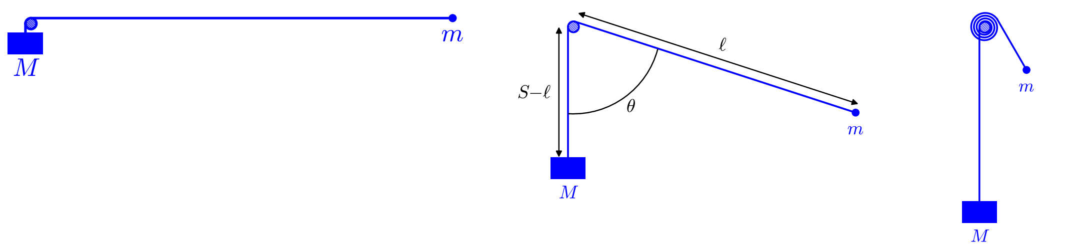
Experimental challenge*: **A not so free fall**

Simple waves — Deadline: 7/March/2018

Experimental part

You have to construct a pulley with a string of length S and a not very thick stick. In the right extreme hang a light mass m and in the left extreme a much heavier mass M. Start with M in the upper position and m with the the string in horizontal position (see the figures below). Release the system to let m oscillate as a pendulum and M fall.



Starting position — Releasing the system — Final situation

The surprise is that with reasonable magnitudes, M stops abruptly because m gives many $360°$ turns and wraps up the string around the stick producing a big friction.

In tests I chose a key as light object and a ball bearing as heavy object. I used a string of something less than $2\ m$. I tried heavier objects and they never reached the floor but I was reluctant to try very heavy objects because my string was very thin.

Mathematical part

Use Euler Lagrange equations with the coordinates ℓ and θ displayed in the second figure to find the differential equations for the evolution of the system with this coordinates assuming no friction and the radius of the pulley 0. Recall that the Lagrangian is $L = \frac{1}{2}Mv_M^2 + \frac{1}{2}mv_m^2 - Mgh_M - mgh_m$ with v velocity and h height.

Write the equations in the form $\ell'' = \dots$, $\theta'' = \dots$ and take formally the limit of them when $m/M \to 0$ (this is physically admissible because m is light and M is heavy) to get a simplified system of equations. From it, find explicitly $\ell(t)$ and get a differential equation for θ not involving ℓ. Try to proof, at least numerically with an example, that $\theta \to -\infty$ when $\ell \to 0^+$. This explains mathematically the experiment because it is known that the friction increases exponentially with the number of turns (Capstan equation) and we have seen that this number goes to ∞, hence the system must stop before $\ell = 0$.

*Some experiments are classical, some are my idea and others come from specific sources. In the latter case I have omitted the reference here on purpose to force the students to work on their own. If you are the author, please do not get angry. I intend to incorporate the references to the final version of the notes.