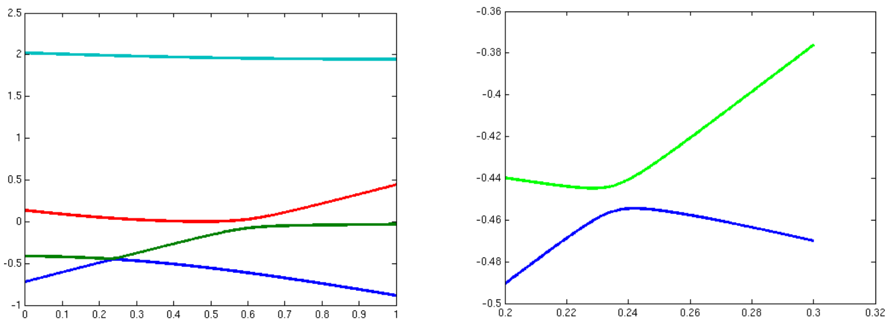
Deadline: September 25th

If your computer chooses two real symmetric $n \times n$ matrices A and B "randomly" then when plotting the eigenvalues of $A + tB$, $0 < t < 1$, you will obtain n curves (you can do it easily with octave or other mathematical packages). Surprisingly, it seems that they avoid crossings. When they are going to match, it seems that they miraculously "bounce".

From time to time the computer shows a possible counterexample (left) but a zoom (right) reveals that it is not.



Probably it is difficult to make a clean theorem out of this (without long explanations about probability spaces, etc.) but the rough idea is that if we have two "random" objects of dimensions n and m in a space of dimension D they usually do not intersect if $n + m < D$. As an example think about linear spaces in $\mathbb{R}^D$. We consider the curve $A + tB$, $0 < t < 1$ as the first object and the set M of $n \times n$ symmetric matrices with a repeated eigenvalue, as the second object. They lie in the space of $n \times n$ symmetric matrices having dimension $D = n(n+1)/2$. Then we can explain the previous phenomenon if $\dim M < n(n+1)/2 - 1$. The question is:

What is the dimension of M?

Of course, the formula is less important, the grade depends on the argument leading to it. Even intuitive arguments contribute to the grade if they are meaningful.

Recall that we saw in class that the dimension is 1 when $n = 2$. The proof is as follows: A symmetric matrix is always diagonalizable. Then if $n = 2$ any element of M is of the form $P^{-1}\lambda I_2 P = \lambda I_2$, with I_2 the 2×2 identity matrix, hence it depends only on a free parameter λ.