COMMENT ON THE COMPUTATION OF THE INDEX IN "HIGHER STEENROD SQUARES IN KHOVANOV HOMOLOGY"

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Abstract. This is a comment on the definition of index in the last paragraph of Section 3.1 in page 21 of the article "Higher Steenrod squares in Khovanov homology"

The definition of *index* in Section 3.1 of [1] has two sources of confusion:

(1) that the subindex i changes depending on whether u_i is considered as an element of $\bar{U}$ or of U. It has to be considered as an element of $\bar{U}$.
(2) that the index function is multivalued on $\bar{U}$, but single-valued in U.

Here is a more detailed procedure to compute the index of an element in a sequence:

Step 1. Compute the index of the elements of $\bar{U}$ (in this case $\bar{U} = 0, 1, 2, 3, 4$):

$$\begin{aligned} \mathrm{ind}_{\bar{U}}(0) &= 0 + 1 \\ \mathrm{ind}_{\bar{U}}(1) &= 1 + 2 \\ \mathrm{ind}_{\bar{U}}(2) &= 2 + 3 \\ \mathrm{ind}_{\bar{U}}(3) &= 3 + 4 \\ \mathrm{ind}_{\bar{U}}(4) &= 4 + 5 \end{aligned}$$

Step 2. To each element in $\ddot{U}$, add the index $\mathrm{ind}_{\bar{U}}(u_i) + 1$.

$$\begin{aligned} \mathrm{ind}_U(0) &= 1 \\ \mathrm{ind}_U(1) &= 3, 4 \\ \mathrm{ind}_U(2) &= 5, 6 \\ \mathrm{ind}_U(3) &= 7 \\ \mathrm{ind}_U(4) &= 9 \end{aligned}$$

Example 1. U = (2,2,6,8,8,9)

$$\begin{aligned} \mathrm{ind}_U(2) &= 3, 4 \\ \mathrm{ind}_U(6) &= 8 \\ \mathrm{ind}_U(8) &= 11, 12 \\ \mathrm{ind}_U(9) &= 13 \end{aligned}$$

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Example 2. U = (0,2,4,4,6,6,8,8)

$$\mathrm{ind}_U(0) = 1$$
$$\mathrm{ind}_U(2) = 4$$
$$\mathrm{ind}_U(4) = 7, 8$$
$$\mathrm{ind}_U(6) = 10, 11$$
$$\mathrm{ind}_U(8) = 13, 14$$

Thanks to Tyler Lawson and Paul Pravakar for remarking the confusion in this definition.

References

[1] Federico Cantero Morán, *Higher Steenrod squares for Khovanov homology*, Advances in Mathematics 369 (2020) 107153 1–79