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Title: Homogenous Riemann surfaces.

Abstract: We are interested in spaces that look the same from any point of the space (that is, "homogeneous spaces"). Of course the notion of looking the same is dependent on the context. For example, the simply connected Riemann surfaces, namely the Riemann sphere, complex plane, and unit disc are conformally homogeneous Riemann surfaces. In fact, along with the punctured plane and the torus these are the only ones. On the other hand, given any surface it is not difficult to cook up a diffeomorphism between any two points of the surface. Hence one needs a notion that is not as strong as conformality and not as weak as differentiability. The key observation is that while smooth maps can distort infinitesimal circles to ellipses with unbounded eccentricity (the ratio of the major to the minor axis can be arbitrarily large), conformal maps do not distort infinitesimal circles at all. This leads to the notion of a homeomorphism being K-quasiconformal (has eccentricity bounded by K). Conformal homeomorphisms are 1-quasiconformal.

A Riemann Surface X is said to be K-quasiconformally homogeneous if for any two points x and y on it, there exists a K-quasiconformal self-mapping taking x to y. If such a K exists we say that X is a QCH Riemann surface. After introducing the basics, the focus of this talk will be on Riemann surface structures that are QCH, and their connections to the topology and hyperbolic geometry of the surface.