

$$d = 4$$

$$\Phi_{\mathbb{Q}}(4) = \left\{ \begin{array}{l} (3), (4), (5), (6), (7), (8), (9), (10), (12), (13), (15), (16), (20), (24), \\ (2, 2), (2, 4), (2, 6), (2, 8), (2, 10), (2, 12), (2, 16), (3, 3), (3, 6), (4, 4), (4, 8), (5, 5), (6, 6) \end{array} \right\}$$

G	$\Phi_{\mathbb{Q}}(4, G) \setminus \{G\} \supseteq$
()	{(3), (5), (7), (9), (13), (15), (3, 3), (5, 5)}
(2)	{(4), (6), (8), (10), (12), (16), (20), (24), (2, 2), (2, 4), (2, 6), (2, 8), (2, 10), (2, 12), (2, 16), (3, 6), (4, 4), (4, 8), (6, 6)}
(3)	{(15), (3, 3)}
(4)	{(8), (12), (16), (24), (2, 4), (2, 8), (2, 12), (2, 16), (4, 4), (4, 8)}
(5)	{(15), (5, 5)}
(6)	{(12), (24), (2, 6), (2, 12), (3, 6), (6, 6)}
(7)	{}
(8)	{(16), (2, 8), (2, 16), (4, 8)}
(9)	{}
(10)	{(20), (2, 10)}
(12)	{(24), (2, 12)}
(2, 2)	{(2, 4), (2, 6), (2, 8), (2, 12), (2, 16), (4, 4), (4, 8)}
(2, 4)	{(2, 8), (2, 16), (4, 4), (4, 8)}
(2, 6)	{(2, 12)}
(2, 8)	{(2, 16), (4, 8)}

$$h_{\mathbb{Q}}(4) \geq 9 \quad \#\mathcal{H}_{\mathbb{Q}}(4) \geq 130 \quad N_{\mathbb{Q}}(4) \geq 14400$$

G	$\mathcal{H}_{\mathbb{Q}}(4, E)$	Label
()	(3)	19a2
()	(5)	11a2
()	(7)	208d1
()	(9)	54a2
()	(13)	2890d1
()	$(3)^2$	121b1
()	(3), (5)	50a2
()	(3), (15)	50b3
()	$(5)^2$	99d2
()	(5), (5, 5)	275b2
()	$(3)^2, (3, 3)$	175b2
()	$(3)^2, (5)$	338d1
()	(3), (5), (15)	50a4
(2)	(2, 2), (2, 4)	200b1
(2)	(4), (2, 2)	46a1
(2)	(4), (2, 6)	36a3
(2)	(4), (2, 10)	450a3
(2)	(4), (2, 2), (2, 4)	49a1
(2)	(4), (2, 2), (2, 10)	1014c2
(2)	(4), (2, 6), (2, 12)	1040g2
(2)	(4), (10), (2, 2)	66c3
(2)	(8), (2, 2), (2, 4)	294f1
(2)	$(4)^2, (2, 2), (2, 4)$	120b1
(2)	$(4)^2, (2, 2), (4, 4)$	320a4
(2)	$(4)^2, (2, 6), (2, 12)$	450g1
(2)	(4), (6), (2, 2), (2, 6)	14a3
(2)	(4), (6), (2, 6), (6, 6)	98a3
(2)	(4), $(6)^2, (2, 2)$	726a2
(2)	(4), (8), (2, 2), (2, 8)	45a1
(2)	(4), (10), (2, 2), (2, 10)	150b3
(2)	(4), (12), (2, 2), (2, 12)	30a3
(2)	(4), (16), (2, 2), (2, 16)	3150bk1
(2)	$(6)^2, (2, 2), (2, 4)$	256a1
(2)	(6), (12), (2, 2), (2, 6)	36a4
(2)	$(8)^2, (2, 2), (4, 8)$	2880r6
(2)	(10), (20), (2, 2), (2, 10)	450a4
(2)	$(4)^2, (2, 2), (2, 4)^2$	33a4

G	$\mathcal{H}_{\mathbb{Q}}(4, E)$	Label
(2)	$(4)^2, (2, 6), (2, 12)^2$	960o7
(2)	$(4)^2, (8), (2, 2), (2, 4)$	33a2
(2)	$(4)^2, (8), (2, 2), (4, 4)$	64a4
(2)	$(4), (6), (2, 2), (2, 4), (2, 6)$	130a4
(2)	$(4), (8), (2, 2), (2, 4), (2, 8)$	144b1
(2)	$(4), (8), (12), (2, 2), (2, 12)$	960e3
(2)	$(4), (8), (16), (2, 2), (2, 8)$	63a1
(2)	$(4), (12), (2, 2), (2, 4), (2, 12)$	720j3
(2)	$(4), (12), (24), (2, 2), (2, 12)$	960o3
(2)	$(4)^2, (8), (2, 2), (2, 4)^2$	17a3
(2)	$(4)^2, (8)^2, (2, 2), (2, 4)$	45a3
(2)	$(4), (8)^2, (2, 2), (2, 4), (2, 8)$	24a6
(2)	$(4), (8)^3, (2, 2), (2, 8)$	45a6
(2)	$(4), (8), (16)^2, (2, 2), (2, 8)$	75b1
(2)	$(4)^2, (8)^2, (2, 2), (2, 4)^2$	63a6
(2)	$(4), (6)^2, (2, 2), (2, 6)^2, (3, 6)$	112c3
(2)	$(4), (8)^3, (16), (2, 2), (2, 8)$	75b6
(2)	$(4), (8)^2, (16), (2, 2), (2, 4), (2, 8)$	510e7
(2)	$(4), (8), (16)^2, (2, 2), (2, 4), (2, 8)$	1470k3
(2)	$(6)^2, (12), (2, 2), (2, 6)^2, (3, 6)$	98a4
(2)	$(4)^2, (6), (12)^2, (2, 2), (2, 4), (2, 6)$	30a7
(2)	$(4)^2, (8)^3, (2, 2), (2, 4)^2$	15a5
(2)	$(4)^2, (8)^4, (2, 2), (2, 4)$	630c6
(2)	$(4)^2, (8)^4, (2, 2), (4, 4)$	4410r6
(2)	$(4), (8)^2, (16)^2, (2, 2), (2, 4), (2, 8)$	1680p1
(2)	$(4)^2, (6), (8), (12)^2, (2, 2), (2, 4), (2, 6)$	90c5
(2)	$(4)^2, (6), (12)^2, (2, 2), (2, 4)^2, (2, 6)$	90c4
(3)	(3, 3)	19a1
(3)	(15)	50a1
(4)	(2, 4), (2, 8)	64a3
(4)	(8), (2, 4)	33a3
(4)	(8), (2, 8)	192c6
(4)	(8), (2, 12)	150c3
(4)	(8), (4, 4)	40a4
(4)	(8), (2, 4), (2, 8)	17a4
(4)	(8), (2, 4), (4, 4)	17a1
(4)	(8), (2, 8), (2, 16)	1470k1

G	$\mathcal{H}_{\mathbb{Q}}(4, E)$	Label
(4)	$(8)^2, (2, 4), (2, 8)$	24a3
(4)	$(8)^2, (2, 8), (4, 8)$	240d6
(4)	$(8), (12), (2, 4), (2, 12)$	90c1
(4)	$(12), (24), (2, 4), (2, 12)$	960o8
(4)	$(8)^2, (16), (2, 4), (2, 8)$	21a4
(4)	$(8)^2, (2, 4), (2, 8)^2, (4, 4)$	195a6
(4)	$(8)^2, (16)^2, (2, 4), (2, 8)$	15a7
(4)	$(8)^2, (16)^3, (2, 4), (2, 8)$	1230f4
(4)	$(8)^2, (16)^2, (2, 4), (2, 8)^2, (4, 4)$	210e6
(5)	$(5, 5)$	11a1
(5)	(15)	50b1
(6)	$(12), (2, 6)$	14a4
(6)	$(12), (2, 6), (2, 12)$	130a2
(6)	$(12), (2, 6), (3, 6), (6, 6)$	14a1
(6)	$(12)^2, (2, 6), (2, 12)$	30a1
(6)	$(12)^2, (2, 6), (2, 12)^2$	90c7
(6)	$(12)^2, (24), (2, 6), (2, 12)$	90c8
(8)	$(16), (2, 8)$	21a3
(8)	$(16), (2, 8), (2, 16)$	1230f1
(8)	$(16), (2, 8), (4, 8)$	15a4
(8)	$(16)^2, (2, 8), (2, 16)$	210e1
(10)	$(20), (2, 10)$	66c1
(12)	$(24), (2, 12)$	90c3
(2, 2)	$(2, 4), (4, 4)$	64a1
(2, 2)	$(2, 4)^3$	33a1
(2, 2)	$(2, 4)^2, (2, 8)$	63a2
(2, 2)	$(2, 4)^2, (2, 12)$	960o6
(2, 2)	$(2, 4)^2, (4, 4)$	17a2
(2, 2)	$(2, 4)^2, (4, 8)$	1200j4
(2, 2)	$(2, 4), (2, 8), (4, 8)$	75b3
(2, 2)	$(2, 4)^3, (2, 6)$	210a6
(2, 2)	$(2, 4)^3, (2, 8)$	45a5
(2, 2)	$(2, 4)^3, (4, 4)$	231a3
(2, 2)	$(2, 4)^2, (2, 6), (2, 12)$	30a6
(2, 2)	$(2, 4)^2, (2, 8), (2, 16)$	75b2
(2, 2)	$(2, 4)^2, (2, 8), (4, 4)$	40a1
(2, 2)	$(2, 4)^2, (2, 8), (4, 8)$	510e5

G	$\mathcal{H}_{\mathbb{Q}}(4, E)$	Label
(2, 2)	$(2, 4), (2, 6), (2, 12)^2$	14400bo6
(2, 2)	$(2, 4)^3, (2, 6), (2, 12)$	90c2
(2, 2)	$(2, 4)^3, (2, 8)^2$	45a2
(2, 2)	$(2, 4)^3, (2, 8), (4, 4)$	21a2
(2, 2)	$(2, 4)^2, (2, 6), (2, 12)^2$	720j6
(2, 2)	$(2, 4)^2, (2, 8)^2, (4, 4)$	75b5
(2, 2)	$(2, 4), (2, 6), (2, 12)^3$	150c6
(2, 2)	$(2, 4)^3, (2, 8)^2, (4, 4)$	42a3
(2, 2)	$(2, 4)^2, (2, 8)^3, (4, 4)$	294c2
(2, 2)	$(2, 4)^3, (2, 8)^3, (4, 4)$	15a2
(2, 2)	$(2, 4)^2, (2, 8)^4, (4, 4)$	6720cd4
(2, 2)	$(2, 4)^3, (2, 8)^4, (4, 4)$	210e5
(2, 4)	$(2, 8)^2, (4, 4)$	21a1
(2, 4)	$(2, 8)^2, (4, 8)$	1230f2
(2, 4)	$(2, 8), (4, 4), (4, 8)$	15a1
(2, 4)	$(2, 8)^2, (2, 16), (4, 4)$	15a3
(2, 4)	$(2, 8)^2, (4, 4), (4, 8)$	210e3
(2, 6)	$(2, 12)^3$	30a2
(2, 8)	$(2, 16)^2, (4, 8)$	210e2