Seminario de Análisis Complejo

Departamento de Matemáticas
Universidad Autónoma de Madrid

Spectral synthesis for systems of exponentials and reproducing kernels

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Abstract: Let $\{x_n\}_{n\in N}$ be a complete and minimal system in a separable Hilbert space H, and let $\{y_n\}$ be its biorthogonal system. The system $\{x_n\}$ is said to be hereditarily complete if for any $x \in H$ we have $x \in \mathrm{Span}\{(x, y_n)x_n\}$. This property can be understood as a very weak form of the reconstruction of a vector x from its (formal) Fourier series $\sum_n (x, y_n)x_n$.

Recently, in a joint work with Yurii Belov and Alexander Borichev we solved the hereditary completeness problem for exponential systems in $L^2(-a, a)$ (equivalently, for systems of reproducing kernels in the Paley-Wiener space PW_a^2). It turned out that in this case the nonhereditary completeness may occur, however the defect of incompleteness is always at most one.

We also study the hereditary completeness for the reproducing kernels in Hilbert spaces of entire functions introduced by L. de Branges. One of our motivations is in the relation (via a functional model obtained jointly with Dmitry Yakubovich) between this problem and the spectral synthesis for rank one perturbations of compact selfadjoint operators. We give a complete description of de Branges spaces where nonhereditarily complete systems of reproducing kernels exist, in terms of their spectral measures. This gives a series of striking examples of (nondissipative) rank one perturbations of compact selfadjoint operators, for which the spectral synthesis fails up to finite- or even infinite-dimensional defect.

The talk is based on joint work with Yurii Belov, Alexander Borichev and Dmitry Yakubovich.

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15:30 h., Módulo **17 (antiguo C-XV)** aula **102**