Composition semigroups on Morrey spaces

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Resumen / Abstract:

A semigroup of analytic functions in the disc is a family $(\varphi_t)_{t \geq 0}$ of analytic functions in the unit disc $D$ with $\varphi_t(D) \subset D$ which satisfy the following conditions:

- $\varphi_0$ is the identity in $D$.
- $\varphi_{t+s} = \varphi_t \circ \varphi_s$, for all $t, s \geq 0$.
- $\varphi_t \to \varphi_0$, as $t \to 0$, uniformly on compact subsets of $D$.

Each such semigroup induces a semigroup $(C_t)$ of composition operators on $H(D)$, the space of analytic functions in $D$,

$$C_t(f) = f \circ \varphi_t, \quad f \in H(D).$$

Given a Banach space $X$ consisting of functions of $H(D)$ and a semigroup $(\varphi_t)$, we say that $(\varphi_t)$ generates a semigroup of operators on $X$ if $(C_t)$ is a well-defined strongly continuous semigroup of bounded operators in $X$. This exactly means that for every $f \in X$, we have $C_t(f) \in X$ for all $t \geq 0$ and

$$\lim_{t \to 0^+} \|C_t(f) - f\|_X = 0.$$

Classical choices of $X$ treated in the literature are the Hardy spaces $H^p$, the disk algebra $A$, the Bergman spaces $A^p$, the Dirichlet space $D$ and the chain of spaces $Q_p$ and $Q_{p,0}$ which include the spaces $BMOA$, Bloch as well as their “little oh” analogues.

In this work we study the action of $(C_t)$ to the class of Morrey spaces

$$L^{2,\lambda} = \left\{ f \in H^2 : \sup_{a \in D} \left(1 - |a|^2\right)^{1-\lambda} \int_D |f'(z)|^2 \left(1 - |\sigma_a(z)|^2\right) dA(z) < \infty \right\},$$

where $\sigma_a(z) = \frac{a-z}{1-\bar{a}z}$, $a \in D$ are the Möbius maps and $0 < \lambda < 1$.

This work is based on a joint work with professors P. Galanopoulos and A. G. Siskakis from Aristotle University of Thessaloniki, Greece.