SEMINARIO DE ANÁLISIS COMPLEJO
(COMPLEX ANALYSIS SEMINAR)

Composition semigroups on Morrey spaces

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Viernes, 19 de octubre 2018 a las 10:30
(Friday, October 19, 2018 at 10:30)

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Resumen / Abstract:

A semigroup of analytic functions in the disc is a family $(\varphi_t)_{t\geq 0}$ of analytic functions in the unit disc $\mathbb{D}$ with $\varphi_t(\mathbb{D}) \subset \mathbb{D}$ which satisfy the following conditions:

- φ_0 is the identity in $\mathbb{D}$.
- $\varphi_{t+s} = \varphi_t \circ \varphi_s$, for all $t, s \geq 0$.
- $\varphi_t \to \varphi_0$, as $t \to 0$, uniformly on compact subsets of $\mathbb{D}$.

Each such semigroup induces a semigroup (C_t) of composition operators on $\mathcal{H}(\mathbb{D})$, the space of analytic functions in $\mathbb{D}$,

$$C_t(f) = f \circ \varphi_t, \quad f \in \mathcal{H}(\mathbb{D}).$$

Given a Banach space X consisting of functions of $\mathcal{H}(\mathbb{D})$ and a semigroup (φ_t), we say that (φ_t) generates a semigroup of operators on X if (C_t) is a well-defined strongly continuous semigroup of bounded operators in X. This exactly means that for every $f \in X$, we have $C_t(f) \in X$ for all $t \geq 0$ and

$$\lim_{t\to 0^+} \|C_t(f) - f\|_X = 0.$$

Classical choices of X treated in the literature are the Hardy spaces H^p, the disk algebra $\mathcal{A}$, the Bergman spaces A^p, the Dirichlet space $\mathcal{D}$ and the chain of spaces Q_p and $Q_{p,0}$ which include the spaces $BMOA$, Bloch as well as their "little oh" analogues.
In this work we study the action of (C_t) to the class of Morrey spaces

$$\mathcal{L}^{2,\lambda} = \left\{ f \in H^2 : \sup_{a\in\mathbb{D}} (1-|a|^2)^{1-\lambda} \int_{\mathbb{D}} |f'(z)|^2 (1-|\sigma_a(z)|^2)\, dA(z) < \infty \right\},$$

where $\sigma_a(z) = \frac{a-z}{1-\bar{a}z}$, $a \in \mathbb{D}$ are the Möbius maps and $0 < \lambda < 1$.
This work is based on a joint work with professors P. Galanopoulos and A. G. Siskakis from Aristotle University of Thessaloniki, Greece.