SHORT COURSE:

Aleksandrov-Clark measures and composition operators

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Wednesday, 16 December, 2015, 15:00 - 16:45

(Aula 520, Módulo 17, Departamento de Matemáticas, UAM)

Summary:

Let ϕ be an analytic map taking the unit disc into itself. Then ϕ determines a family of positive measures τ_α supported on the unit circle and defined by the Poisson integral

$$\mathrm{Re}\frac{\alpha+\phi(z)}{\alpha-\phi(z)} = \int P_z \, d\tau_\alpha$$

for every α on the unit circle. These measures were introduced by D.N. Clark (1972) in a spectral-theoretic context and many of their deep function-theoretic properties were later established by A.B. Aleksandrov (1987).

We start by examining basic function-theoretic properties of the family τ_α; in particular, the correspondence between the atoms of τ_α and the angular derivatives of ϕ. We also discuss Aleksandrov's disintegration formula for the Lebesgue measure.

The Aleksandrov-Clark measures induce a natural integral operator for functions defined on the unit circle. This operator, often called the Aleksandrov operator, has very nice properties as it preserves many important function spaces. It also turns out to have an intimate connection to the composition operator $f \mapsto f \circ \phi$ induced by ϕ. This makes it possible to explain the operator-theoretic properties of composition operators in terms of the Aleksandrov-Clark measures.