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SOME OLD AND SOME NEW THOUGHTS ON COMMUTANTS OF ANALYTIC MULTIPLICATION OPERATORS

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Except in special circumstances, it is usually quite difficult to determine conditions that characterize which operators commute with a given operator. Such special circumstances include use of the spectral theorem for self-adjoint or normal operators and cases in which the operator in question has a rich point spectrum. The results in this latter situation come from the application of the fairly trivial observation that if A and B commute, the eigenspaces of A are invariant for B.

If $\mathcal{H}$ is a Hilbert space of analytic functions on the unit disk and T_z is the operator of multiplication by z, it is well known that the commutant of T_z is the collection of multiplication operators T_f where f is a bounded analytic function on the disk and $(T_f h)(z) = f(z)h(z)$.

In the 1970's and 80's, the question "Which operators on the Hardy space $H^2(\mathbb{D})$ commute with T_f for f a bounded analytic function on the disk?" was investigated. More recently, there has been interest in this question for the Bergman space $A^2(\mathbb{D})$. In this talk, an overview of the work of thirty years ago will be presented and we will consider this question for $f = B$, a finite Blaschke product, for T_B acting on the Bergman space. The question has wider consequences than might be expected and the answer is given in terms of a Riemann surface associated with the Blaschke product.