SHORT COURSES

Transversal multilinear harmonic analysis

Jonathan Bennett
Birmingham University, UK
j.bennett@bham.ac.uk

Abstract

Many important problems in multivariable euclidean harmonic analysis concern the control of a linear operator whose subtle properties are due to the presence of some underlying curved manifold (this is a popular and enduring perspective made explicit in work of Stein and Wainger from 1978). In some situations such operators have multilinear counterparts where the role of curvature is replaced by a notion of transversality. The main focus of these lectures will be the recent multilinear approach to Stein's celebrated restriction conjecture for the Fourier transform. In particular, the d-linear restriction problem will be discussed in some detail, including its relation with the classical linear problem via the 2010 method of Bourgain and Guth. We conclude with a description of a bigger picture which encompasses oscillatory integrals of Hörmander type and a broad class of multilinear Radon-like transforms that arise in dispersive PDE.

Aspects of harmonic analysis related to hypersurfaces and Newton diagrams

Detlef Müller
C.A.-Universität Kiel, Germany
mueller@math.uni-kiel.de

Abstract

For several decades now, a major part of research in Euclidean harmonic analysis is driven by the problem to understand the interplay between geometric properties, such as curvature properties of certain subvarieties, and related operators, such as maximal averaging operators along hypersurafces, Bochner-Riesz-multipliers, various classes of oscillatory integral operators, or (singular) Radon transforms, to give just a few examples. Important instances are Stein's spherical maximal operator, or Fourier restriction operators to spheres or cones. Naturally, a lot of attention has been given to the most generic situations, for instance hypersurfaces with non-vanishing curvature, for which deep, but in general still not complete results have been obtained.

However, what can be said about more general varieties, with weaker geometric properties? For some of these problems, related to smooth, finite type hypersurfaces S in $\mathbb{R}^3$, in joint work with I. Ikromov we have been able to give essentially complete answers in terms of Newton diagrams associated to the given hypersurface. More precisely, if we denote by $d\mu = \rho d\sigma$ a surface carried measure with smooth, compactly supported density $\rho \geq 0$ with respect to the surface measure $d\sigma$, then I shall discuss in particular the following questions:

A. Find the best possible uniform decay estimates for the Fourier transform $\hat{\mu}$ of the surface carried measure $d\mu$.

B. Determine the range of exponents p for which a Fourier restriction estimate

$$\Big(\int_S |\hat{f}(x)|^2 \, d\mu(x)\Big)^{1/2} \leq C\|f\|_{L^p(\mathbb{R}^3)}$$

holds true. The first problem is classical, and the second problem has been introduced by E.M. Stein. Indeed, owing a lot to the insight of Arnol'd and his school, it is known that the first question can be answered by means of Newton diagrams when S is analytic, and we have shown that it is possible to extend this to the non-analytic case.

The program of my talks will thus comprise a review of various important notions, such as Newton polyhedra, adaptedness of a given coordinate system, etc.. Subsequently I shall try to give an idea how properties of the Newton polyhedra attached to the surface will reflect the decay rate of $\hat{\mu}(\xi)$, and finally I shall address question B and indicate some major ideas used in the proof of our main results. We shall see in particular that problems A and B are not as directly related as one might perhaps expect.

Analysis on non-smooth domains

Tatiana Toro
University of Washington
toro@uw.edu

Abstract

A central theme in potential theory is understanding the extent to which the geometry of a domain influences the boundary regularity of solutions to divergence form elliptic operators. On regular domains (in Wiener's sense) one can associate to such operators a family of probability measures indexed by the points in the domain. All measures in this family are mutually absolutely continuous (in fact they are A_∞ weights with respect to one another). One refers to any one of them as the elliptic measure. To address the question of boundary regularity of the solutions of these operators one studies the properties of the corresponding elliptic measure. This has lead to the development of harmonic analysis techniques on non-smooth domains. This is achieved by combining ideas from classical harmonic analysis with techniques from geometric measure theory.

In this mini-course we will discuss the Dirichlet problem on several types of non-smooth domains (e.g. Reifenberg flat domains, chord arc domains, chord arc domains with small constant). We will discuss to extent to which the geometry of the domain determines the properties of the elliptic measure. We will also discuss the converse problem namely, the degree to which the regularity of the elliptic measure determines the geometry of the domain.

Estimating the maximal singular integral in terms of the singular integral

Joan Verdera

Universitat Autònoma de Barcelona

joan.verdera@uab.cat

Abstract

We consider the problem of estimating the maximal singular integral T^*f in terms of Tf only. The problem arose when I was working on a variation of the David-Semmes problem, consisting in deriving existence of principal values from L^2 boundedness for certain particular Calderón-Zygmund operators defined on subsets of $\mathbb{R}^n$. Concretely, we shall study the validity of the $L^2(\mathbb{R}^n)$-inequality $\|T^*f\|_2 \leq C\,\|Tf\|_2$ in the context of classical Calderón-Zygmund Theory, namely, for operators defined by a principal value convolution $Tf(x) = p.v. \int f(x-y)\,K(y)\,dy$ where the kernel is of the form $K(x) = \Omega(x)/|x|^n$, $x \in \mathbb{R}^n \setminus \{0\}$, with Ω homogeneous of degree 0, of class C^∞ on the unit sphere and with zero integral there. For example, $\Omega(x)$ may be taken to be $P(x)/|x|^d$ where P is a harmonic homogeneous polynomial of degree d. In that case T is called a higher Riesz transform. It turns out that for an even higher Riesz transform a pointwise inequality stronger than the L^2-estimate holds : $T^*f(x) \leq C\,M(Tf)(x)$, $x \in \mathbb{R}^n$. Notice that this is a half the well-known Cotlar's inequality $T^*f(x) \leq C\,(M(Tf)(x) + M(f)(x))$, $x \in \mathbb{R}^n$, which is of no use for us because the dependence on f does not come through Tf only. For odd higher Riesz transforms the above inequality fails (even for the Hilbert transform). One has the weaker result $T^*f(x) \leq C\,M^2(Tf)(x)$, $x \in \mathbb{R}^n$, where M^2 is the iteration of the maximal operator.

In joint work with Mateu, Orobitg and Carlos Pérez we have described those T of a given parity for which the L^2-inequality holds. The equivalent condition is stated in terms of the spherical harmonics expansion of Ω and is algebraic. It is also equivalent to the appropriate pointwise estimate mentioned above. In the first two lectures I plan to present the proof of the pointwise inequality for even higher Riesz transforms (and say some words on the odd case) and discuss how one gets necessary conditions for the L^2 estimate in a particularly simple situation. A description on what one has to do in the general case will be sketched. When considering the sufficient condition a tangential contact with 2 dimensional fluid dynamics (vortex patches) arises. In the third lecture I will deal with the basics of vortex patches, showing how the Cauchy and the Beurling transforms enter the scene and I will mention a boundary regularity result for rotating vortex patches we have recently obtained.

MAIN SPEAKERS

Multi-frequency Calderón-Zygmund analysis applied to Bochner-Riesz multipliers

Frédéric Bernicot
CNRS - Université de Nantes, France
frederic.bernicot@univ-nantes.fr

Abstract

We plan to present a "multi-frequency Calderón-Zygmund analysis", introduced by Nazarov, Oberlin and Thiele and some applications. These results extend the classical theory. Indeed, it allows us to study a new kind of Calderón-Zygmund operators, mainly the sum of modulated Calderón-Zygmund operators.

Aiming to extend the classical results to these new operators, we first prove boundedness and then weighted boundedness. To do so, we make use of the abstract good-λ inequalities of Auscher and Martell, associated to a maximal sharp function (taking care of the "multi-frequency" framework).

This work is motivated by applications to the well-known Bochner-Riesz multipliers. Indeed, we will explain how such arguments allow us to get new weighted boundedness for such multipliers, involving Muckenhoupt's weights. Moreover, this approach is very general and can be applied to generalised Bochner-Riesz multipliers (where the disc or the ball is replaced by another geometrical set).

Existence of frames with prescribed norms and frame operator

Marcin Bowník
University of Oregon, U.S.A.
mbownik@uoregon.edu

Abstract

In this talk we present several recent results on the existence of frames with prescribed norms and frame operator. These results are equivalent to the Schur-Horn type theorems which describe possible diagonals of positive self-adjoint operators with specified spectral properties. The first infinite dimensional result of this type is due to Kadison who characterized diagonals of orthogonal projections. Kadison's theorem gives automatically a characterization of all possible sequences of norms of Parseval frames. We present some generalizations of Kadison's result such as: (a) the lower and upper frame bounds are specified, (b) the frame operator has 2 point spectrum, (c) the frame operator has a finite spectrum. This talk is based on the joint work with John Jasper.

Finite time singularities for incompressible fluids

Diego Córdoba
Instituto de Ciencias Matemáticas-CSIC, Spain
dcg@icmat.es

Abstract

We consider the evolution of an interface generated between two immiscible, incompressible and irrotational fluids. Specifically we study the Muskat equation (the interface between oil and water in sand) and the water wave equation (interface between water and vacuum). For both equations we show the existence of smooth initial data for which the smoothness of the interface breaks down in finite time. Joint work with A. Castro, C. Fefferman, F. Gancedo, J. Gómez-Serrano and M. López-Fernández.

Smoothing for the KdV equation and the Zakharov system on the torus

Burak Erdogan
University of Illinois at Urbana Champaign, USA
berdogan@math.uiuc.edu

Abstract

In this talk we will consider the periodic KdV equation and the periodic Zakharov system in one dimension. We prove that the difference of the nonlinear and the linear evolutions is in a smoother space than the initial data. The method is based on normal form calculations and $X^{s,b}$ space estimates. We will also discuss applications such as almost everywhere convergence to initial data, growth bounds for higher order sobolev norms, and the existence and smoothness of global attractors. This is a joint work with Nikos Tzirakis.

Pointwise convergence of vector-valued Fourier series

Tuomas Hytönen
University of Helsinki, Finland
tuomas.hytonen@helsinki.fi

Abstract

I report on recent joint work with M. Lacey, where we obtain a vector-valued version of Carleson's theorem: Let Y be a complex interpolation space between a UMD space X and a Hilbert space H. Then the Fourier series of Y-valued functions converge pointwise almost everywhere.

Apparently, all known examples of UMD spaces are intermediate spaces as described. In particular, this covers the noncommutative L^p spaces, and hence we answer affirmatively a question raised by J. L. Rubio de Francia in the 1980's on the convergence of Fourier series of Schatten class valued functions.

The proof consists of extending the Lacey–Thiele approach to time-frequency analysis to the vector-valued setting. We have also considered other related questions in this set-up, including the convergence of vector-valued Walsh series and, jointly with I. Parissis, a vector-valued Walsh model for the bilinear Hilbert transform.

On the cone multiplier in $\mathbb{R}^3$

Sanghyuk Lee
Seoul National University, Korea
shklee@snu.ac.kr

Abstract

We prove the sharp L^3 bounds for the cone multiplier in $\mathbb{R}^3$ and the associated square function, which is known as Mockenhaupt's square function. The results are obtained by an adaptation of recent Bourgain-Guth argument for restriction estimates which relies on the multilinear restriction theorem due to Bennett-Carbery-Tao. This is a joint work with Ana Vargas.

An H_1-BMO duality for semigroups of operators

Tao Mei
Wayne State University
mei@wayne.edu

Abstract

Let (T_t) be an abstract Markov diffusion semigroup of operators on a measure space. We may define a BMO norm associated with (T_t) as $\|f\|_{BMO} = \sup_t \|T_t|f - T_t f|^2\|_\infty^{\frac{1}{2}}$, viewing T_t as an alternative of the mean value operator. The question is how to define an corresponding H_1 norm and to establish Fefferman-Stein's duality theory in this abstract setting, and what conditions on (T_t) are needed? We assume there is no direct information available on the local structure of the underlying measure space and seek for a duality theory relies merely on the semigroups of operators.

A main motivation of this question is from our recent work on noncommutative Fourier multipliers, jointly with Junge and Parcet, where we have extensively used the abstract BMO norm given above in developing a Calderón-Zygmund theory on noncommutative L_p spaces. A main difficulty in the research is to find the "right" noncommutative alternatives to the geometric properties/tools used in the classical analysis.

In this talk, I will limit myself in the setting described in the first paragraph and report recent progress in seeking the desired duality theory by considering H_1 norms defined by semigroup-analogues of Lusin area integrals.

End-point boundary value problems in rough domains

Marius Mitrea
University of Missouri, USA
mitream@missouri.edu

Abstract

Lately, considerable success has been registered in the implementation of the method of layer potentials in the treatment of boundary value problems in classes of domains whose boundaries may not be locally described as graphs. In my talk I will report on such recent progress for elliptic PDE's with special emphasis on the Neumann problem with data in Hardy spaces, and the Dirichlet problem with data in BMO.

These basic BVP's are considered in the class of chord-arc domains with an appropriately small constant, and some of the key issues underscoring the solvability of the aforementioned problems are: a suitable grand-maximal function characterization of Hardy spaces in rough settings, an extension of the classical De Georgi-Federer Divergence theorem, a suitable version of Fredholm theory for boundary integral operators of potential type in Hardy and BMO spaces, and square-function estimates on uniformly rectifiable sets.

Various portions of the work reported on are joint collaborations.

A sharp bilinear estimate for the Klein-Gordon equation in two space-time dimensions

Tohru Ozawa
Waseda University, Japan
txozawa@waseda.jp

Abstract

This talk is based on my recent joint-work with Keith Rogers. We prove a sharp bilinear estimate for the Klein-Gordon equation in two space-time dimensions. We also prove new estimates for the restriction of the Fourier transform to the whole hyperbola, which is noncompact and has a vanishing curvature at infinity.

Weighted Inequalities and dyadic harmonic analysis

Cristina Pereyra
University of New Mexico, USA
crisp@math.unm.edu

Abstract

We survey the recent solution of the so-called A_2 conjecture, all Calderón-Zygmund singular integral operators are bounded on $L^2(w)$ with a bound that depends linearly on the A_2 characteristic of the weight w, as well as corresponding results for commutators. We highlight the interplay of dyadic harmonic analysis in the solution of the A_2 conjecture, especially Hytönen's representation theorem for Calderón-Zygmund singular integral operators in terms of Haar shift operators.

Cauchy non-integral formulas

Andreas Rosén
Linköping University, Sweden
andreas.rosen@liu.se

Abstract

This talk concerns generalizations of the Cauchy integral formula which appear in recent joint work with Pascal Auscher. We study solutions to second order divergence form elliptic systems by representing the gradient vector, and conjugate systems, of the solution as a Cauchy type "integral" of the boundary trace. Besides being bounded and accretive, the coefficients of the equation are only assumed to be close to coefficients that are independent of the variable transversal to the boundary in a Carleson sense defined by Dahlberg. We shall use recent Carleson duality results from joint work with Tuomas Hytönen, to give a simplified presentation of the theory as compared to that in the two papers "Weighted maximal regularity estimates and solvability of non-smooth elliptic systems".

For general coefficients as above, the Cauchy formulas appearing are not integral operators. However, for real scalar equations with coefficients independent of the variable transversal to the boundary, the Cauchy formula is at least partly integral, as it contains the classical double layer potential operator. This gives new boundedness results for this operator, for general coefficients, answering a question posed by Steve Hofmann in El Escorial 2008.

Quasi-greedy bases and Lebesgue-type inequalities

Vladimir Temlyakov
University of South Carolina, USA; Steklov Institute of Mathematics, Russia
temlyakovusc@gmail.com

Abstract

We study Lebesgue-type inequalities for greedy approximation with respect to quasi-greedy bases. We mostly concentrate on this study in the L_p spaces. The novelty is in obtaining better Lebesgue-type inequalities under extra assumptions on a quasi-greedy basis than known Lebesgue-type inequalities for quasi-greedy bases. We consider uniformly bounded quasi-greedy bases of L_p, $1 < p < \infty$, and prove that for such bases an extra multiplier in the Lebesgue-type inequality can be taken as $C(p)\ln(m+1)$. The known magnitude of the corresponding multiplier for general (no assumption of uniform boundedness) quasi-greedy bases is of order $m^{|\frac{1}{2}-\frac{1}{p}|}$, $p \neq 2$. For uniformly bounded orthonormal quasi-greedy bases we get further improvements replacing $\ln(m+1)$ by $(\ln(m+1))^{1/2}$.

Multiwave Inverse Problems

Gunther Uhlmann
University of California Irvine and University of Washington, USA
crisp@math.unm.edu

Abstract

We will consider several inverse problems that involve using different types of waves, coupled trough a physical principle, to determine the internal properties of a medium. Examples of this are Photacoustic and Thermoacoustic Tomography and Transient Elastography.

Performance of greedy algorithm in reduced basis method

Przemysław Wojtaszczyk
University of Warsaw, Poland
wojtaszczyk@mimuw.edu.pl

Abstract

The reduced basis method was introduced for the accurate online evaluation of solutions to a parameter dependent family of elliptic partial differential equations. Abstractly, it can be viewed as determining a "good" n dimensional space $\mathcal{H}_n$ to be used in approximating the elements of a compact set $\mathcal{F}$ in a Hilbert or Banach space $\mathcal{H}$ where solutions live. One, by now popular, computational approach is to find $\mathcal{H}_n$ through a greedy strategy. It is natural to compare the approximation performance of the $\mathcal{H}_n$ generated by this strategy with that of the Kolmogorov widths $d_n(\mathcal{F})$ since the latter gives the smallest error that can be achieved by subspaces of fixed dimension n. The first such comparisons, given in A. Buffa, Y. Maday, A.T. Patera, C. Prud'homme, and G. Turinici, *A Priori convergence of the greedy algorithm for the parameterized reduced basis* M2AN Math. Model. Numer. Anal., **46**(2012), 595–603, show that the approximation error in a Hilbert space, $\sigma_n(\mathcal{F}) := \operatorname{dist}(\mathcal{F}, \mathcal{H}_n)$, obtained by the greedy strategy satisfies $\sigma_n(\mathcal{F}) \leq Cn2^n d_n(\mathcal{F})$. In this talk, various improvements of this result will be given both in Hilbert and in Banach space case. We discuss both individual comparison between $\sigma_n(\mathcal{F})$ and $d_s(\mathcal{F})$ and the estimates for classes when we assume certain decay of $d_n(\mathcal{F})$ and obtain related decay of $\sigma_n(\mathcal{F})$.

This talk reports the joint work with Peter Binev, Albert Cohen, Wolfgang Dahmen, Ronald DeVore and Guergana Petrova.