Short talks. El Escorial 2012.

Alfonseca Cubero, M. Ángeles (North Dakota State University, USA):
Determining convex bodies from their sections.

Abstract: A recurring question in convex analysis is the determination of a convex body K from a given set of its lower dimensional sections or projections. In this talk, we will review several problems about the determination of K from its hyperplane sections, and present some recent results. Different problems arise depending on the type of sections considered (central sections, maximal sections, sections tangent to a prescribed ball inside of K, etc.) In all cases, the main questions are the following:

1. If all the sections of K have the same volume, is K a ball?
2. If all the sections of K have the same volume as the corresponding sections of L, is $K = L$?

Apraiz, Jone (Universidad del País Vasco, Spain):
Null-control and measurable sets.

Abstract: The control for evolution equations aims to drive the solution to a prescribed state starting from a certain initial condition. One acts on the equation through a source term, a so-called distributed control, or through a boundary condition. To achieve general results one wishes for the control to only act in part of the domain or its boundary and to have as much latitude as possible in the choice of the control region: location, size, shape.

We prove the interior and boundary null-controllability of some parabolic evolutions with controls acting over measurable sets. This is a joint work with Luis Escauriaza from Universidad del País Vasco.

Arrizabalaga, Naiara (University of the Basque Country, Spain):
Dirac operator with Coulomb type potentials.

Abstract: In this talk we show recent progress on the study of the Dirac operator with Coulomb type singularities. We prove a partial differential equation related to the operator and give an essential self-adjointness result of the Dirac operator associated to Hermitian matrix potentials with Coulomb decay. We obtain those results by means of a new and interesting Hardy-Dirac inequality which is sharp. In particular, we obtain results for some electromagnetic potentials such that both, the electric potential and the magnetic one, have Coulomb type singularity.

(Joint work with Javier Duoandikoetxea and Luis Vega.)

Beltiţă, Ingrid (Institute of Mathematics of the Romanian Academy, Romania):
Weyl-Pedersen calculus on coadjoint orbits of nilpotent Lie groups.

Abstract: The Weyl-Pedersen calculus we are dealing with is the remarkable correspondence $a \mapsto \mathrm{Op}^{\pi}(a)$ constructed by N.V. Pedersen in [Matrix coefficients and a Weyl correspondence for nilpotent Lie groups. *Invent. Math.* **118** (1994), no. 1, 1–36] as a generalization of the pseudo-differential Weyl calculus on $\mathbb{R}^n$. Here $\pi\colon G \to \mathbb{B}(\mathcal{H})$ is any unitary irreducible representation of a connected, simply connected, nilpotent Lie group G, the symbol a can be any tempered distribution on the coadjoint orbit $\mathcal{O}$ corresponding to π by the orbit method and $\mathrm{Op}^{\pi}(a)$ is a linear operator in the representation space $\mathcal{H}$, which is in general unbounded.

We present here boundedness and compactness properties for the operators obtained by the Weyl-Pedersen calculus in the case of the irreducible unitary representations of nilpotent Lie groups that are associated with flat coadjoint orbits. We use spaces of smooth symbols satisfying appropriate growth conditions expressed in terms of invariant differential operators on the coadjoint orbit under consideration. Our

method also provides conditions for these operators to belong to one of the Schatten ideals of compact operators. In the special case of the Schrödinger representation of the Heisenberg group we recover some classical properties of the pseudo-differential Weyl calculus, as the Calderón-Vaillancourt theorem, and the Beals characterization in terms of commutators.

Bez, Neal (University of Birmingham, United Kingdom):
Optimal constants and extremisers for some smoothing estimates.

Abstract: We present some new results concerning the optimal constant and existence of extremisers for a broad class of smoothing estimates of the form

$$\|\psi(|\nabla|)\exp(it\phi(|\nabla|)f\|_{L^2(w(|x|)\mathrm{d}x\mathrm{d}t)} \leq C\|f\|_{L^2}.$$

Such estimates are of course equivalent to the L^2-boundedness of certain oscillatory integral operators S depending on (w, ψ, ϕ). When the radial weight w is homogeneous, we provide a full spectral decomposition of S^*S and consequently obtain an explicit formula for the optimal constant C and a characterisation of extremisers. In certain well-studied cases when w is inhomogeneous, we obtain new expressions for the optimal constant. This builds on earlier work B. Walther and B. Simon, and is joint work with M. Sugimoto (Nagoya).

Bongioanni, Bruno (Instituto de Matemática Aplicada del Litoral, CONICET-UNL, Argentina):
Extrapolation for classes of weights related to a family of operators.

Abstract:
In this work we deal with the extrapolation property for classes of weights that arise from the L^p boundedness of a one-parameter family of maximal operators, rather than a single operator like in the case of Muckenhoupt classes A_p. The starting point for the extrapolation can be the knowledge of boundedness on a particular Lebesgue space as well as the boundedness on the extremal case L^∞ with some BMO type inequalities. Our approach models the situation of weights appearing in the context of the analysis related to the Schrödinger operator $-\Delta + V$ (see [BHS]). We apply the extrapolation results to obtain weighted scalar and vector valued inequalities for some operators associated to $-\Delta + V$.

[BHS] B. Bongioanni, E. Harboure and O. Salinas. *Classes of weights related to Schrödinger operators*, J. Math. Anal. Appl., 373 (2011), no. 2, 563-579.

Buschenhenke, Stefan (Christian-Albrechts-Universität zu Kiel, Germany):
A sharp L_p-L_q−Fourier restriction theorem for a conical surface of finite type.

Abstract: The curve of finite type $\gamma = \{(x, x^m)|x \in [0, 1]\}$, $m \geq 3$ generates a compact piece of the generalized cone $\Gamma = \{(\xi, z) \in \mathbb{R}^2 \times \mathbb{R}| \, 1 \leq z \leq 2, \ \frac{\xi}{z} \in \gamma\}$ with surface measure σ. Then it is known that
THEOREM (Barceló 1986). *Let $1 \leq p, q \leq \infty$ satisfy $p' \geq 2m$ and $\frac{1}{q} \geq \frac{m+1}{p'}$. Then*

$$\|\hat{f}|_\Gamma\|_{L_q(\Gamma,\sigma)} \leq C\|f\|_{L_p(\mathbb{R}^3)}, \quad \forall \, f \in \mathcal{S}(\mathbb{R}^3). \tag{1}$$

A necessary condition for (1) is: $\quad p' > m + 1$ and $\frac{1}{q} \geq \frac{m+1}{p'}$. (2)
Indeed, in the same paper, this condition is shown to be sufficient for the corresponding Fourier restriction estimate on the curve γ.
I rediscovered Barceló's theorem and showed that the necessary condition (2) is also sufficient for the restriction estimate (1), closing the gap between the two conditions.
The general approach is loosely based on a previous paper of Barceló from 1985 (dealing with the case $m = 2$ with the sharp condition $p' > 4$), but the decomposition of Γ is quite different from Barceló's proof.

Cho, Yonggeun (Chonbuk National University, Republic of Korea):
Strichartz estimates in spherical coordinates.

Abstract: In this talk we will consider Strichartz estimates of linear dispersive equation in spherical coordinates. Under radial assumption for the phase functions a sharp endpoint estimate will be discussed. For this we use spherical harmonic expansion, asymptotic behavior of bessel functions and time-space-frequency localized estimate of oscillatory integral.

Chousionis, Vasileios (Universitat Autonoma de Barcelona):
Homogeneous kernels and rectifiability in the plane.

Abstract: Let $E \subset \mathbb{C}$ be a Borel set with finite length. By a theorem of David and Léger, the $L^2(E)$-boundedness of the singular integral associated to the Cauchy kernel (or even to one of its coordinate parts $x/|z|^2, y/|z|^2, z = (x, y) \in \mathbb{C}$) implies that E is rectifiable. We extend this result to any kernel of the form $x^{2n-1}/|z|^{2n}, z = (x, y) \in \mathbb{C}, n \in \mathbb{N}$. We thus provide the first non-trivial examples of operators not directly related with the Cauchy transform whose $L^2(E)$ -boundedness implies rectifiability. We will also discuss the more delicate case of harmonic homogeneous kernels with zeros.

This is a joint work with J. Mateu, L. Prat and X. Tolsa.

Chua, Seng-Kee (National University of Singapore):
A Compact Embedding Theorem for Generalized Sobolev Spaces.

Abstract: We give an elementary proof of a compact embedding theorem in abstract Sobolev spaces. The result is first presented in a general context and later specialized to the case of degenerate Sobolev spaces defined with respect to non-negative quadratic forms on R^n. Although our primary interest concerns degenerate quadratic forms, our result also applies to nondegenerate cases, and we consider several such applications, including the classical Rellich-Kondrachov compact embedding theorem and results for the class of s-John domains in R^n, the latter for weights equal to powers of the distance to the boundary. Joint work with Scott Rodney and Richard L. Wheeden

Curbera, Guillermo P. (University of Sevilla, Spain):
Khintchine inequality: local and weighted versions.

Abstract: 1. In 1935 Zygmund proved a local version of Khintchine inequality: For any measurable set $E \subset [0, 1]$ with $m(E) > 0$ there exists $N := N(E)$ such that

$$(1-\varepsilon)\sum_{n=N}^{\infty} a_k^2 \le \frac{1}{m(E)}\int_E \Big|\sum_{n=N}^{\infty} a_k r_k(t)\Big|^2 dt \le (1+\varepsilon)\sum_{n=N}^{\infty} a_k^2$$

for every $(a_k) \in \ell^2$. We give a precise characterization of those rearrangement invariant spaces where the local version of Khintchine inequality holds.

2. In 2010 M. Veraar gave a weighted version of Khintchine inequality in $L^p([0, 1])$: Let $1 \le p < \infty$ and $w \in L^q([0, 1])$, for some $q > p$, with $m(\mathrm{supp}(w)) > 2/3$. There exists $C_1, C_2 > 0$ such that

$$C_1\Big(\sum_{i=1}^{\infty} a_i^2\Big)^{1/2} \le \left(\int_0^1 \Big|\sum_{i=1}^{\infty} a_i r_i(t)\Big|^p |w(t)|^p\, dt\right)^{1/p} \le C_2\Big(\sum_{i=1}^{\infty} a_i^2\Big)^{1/2},$$

for every $(a_i) \in \ell^2$. We consider the result for general rearrangement invariant spaces and for weights with support having arbitrary small measure. Collaboration with Sergey V. Astahskin from the University of Samara in Russia.

Dindoš, Martin (University of Edinburgh, UK):
The Boundary value problems for second order elliptic operators satisfying Carleson condition.

Abstract: In 2007 paper with S. Petermichl and J. Pipher we established solvability of the Dirichlet boundary value problem for second order divergence form equations $\text{div}(A\nabla u) = 0$ on Lipschitz domains with boundary values in L^p. In this paper we assumed that the coefficients of the second order operator to be rough obeying a certain natural Carleson condition. It turns out that p for which the boundary value problem is solvable depends on the size of the Carleson norm of coefficients and the Lipschitz norm of the boundary.

Recently with J. Pipher and D. Rule we have considered other natural boundary value problems under same assumption on coefficients of the operator. In particular we have established solvability of the Dirichlet regularity problem with boundary data in $H^{1,p}$ and of the Neumann problem with L^p data. It turns out these boundary value problems are considerable harder than the L^p Dirichlet boundary value problem and until now the result was known only in 2d (2010). We rely significantly on several recent advances that improved our understanding of the Dirichlet and Regularity boundary values problems in particular the "duality" between the solvability of the Dirichlet boundary value problem and the Regularity problem for the adjoint operator.

Di Plinio, Francesco (Indiana University Bloomington, USA):
Logarithmic L^p bounds for maximal directional singular integrals in the plane.

Abstract: Let K be a Calderon-Zygmund convolution kernel on $\mathbb{R}$. We discuss the L^p-boundedness of the maximal directional singular integral

$$T_{\mathbf{V}} f(x) = \sup_{v \in \mathbf{V}} \Big| \int_{\mathbb{R}} f(x+tv) K(t)\, \mathrm{d}t \Big|$$

where $\mathbf{V}$ is a finite set of N directions. Logarithmic bounds (for $2 \leq p < \infty$) are established for a set $\mathbf{V}$ of arbitrary structure. Sharp bounds are proved for lacunary and Vargas sets of directions. The latter include the case of uniformly distributed directions and the finite truncations of the Cantor set.

We make use of both classical harmonic analysis methods and product-BMO based time-frequency analysis techniques. As a further application of the latter, we derive an L^p almost orthogonality principle for Fourier restrictions to cones. This is joint work with Ciprian Demeter.

Dooley, Anthony (University of New South Wales, Australia)
Heat kernels and Brownian motion on compact Lie groups .

Abstract: Let G/K be a Riemannian symmetric space of the compact type, with tangent space at the identity $\mathfrak{p}$. I show how to wrap Brownian motion from $\mathfrak{p}$ onto G/K, thus finding the heat kernel from the Euclidan heat kernel. This method also works for other Lévy processes.

Fariña, Juan Carlos (University of La Laguna, Spain):
Littlewood-Paley functions associated with the Hermite operator and γ-radonifying operators.

Abstract: This talk is based on joint works with J. Betancor, A.J. Castro, and L. Rodríguez-Mesa (University of La Laguna) and J. Curbelo (ICMAT, MAdrid). We study Littlewood-Paley-Stein functions associated with the Poisson semigroup for the Hermite operator $\mathfrak{H}$ with values in a UMD Banach space $\mathbb{B}$. I we denote by H the Hilbert space $L^2((0,\infty), dt/t)$, $\gamma(H,\mathbb{B})$ represents the space of γ-radonifying operators from H into $\mathbb{B}$. We prove that the Hermite square function gives equivalent norms in $L^p(\mathbb{R}^n, B)$, $1 < p < \infty$, $BMO_{\mathfrak{H}}(\mathbb{R}^n, B)$, and $H^1_{\mathfrak{H}}(\mathbb{R}^n, B)$. Also, we obtain new characterization of the UMD Banach spaces by using square functions and γ-radonifying operators.

Grau de la Herran, Ana (University of Missouri-Columbia, USA):
T1 Theorem and local Tb Theorems for Square functions.

Abstract: The Tb theorem, like its predecessor, the T1 Theorem, is a an L^2 boundedness criterion, originally established by McIntosh and Meyer, and by David, Journe and Semmes in the context of singular integrals, but later extended by Semmes to the setting of "square functions". The latter arise in many applications in complex function theory and in PDE, and may be viewed as singular integrals taking values in a Hilbert space. The essential idea of Tb and T1 type theorems, is that they reduce the question of L^2 boundedness to verifying the behavior of an operator on a single test function b (or even the constant function 1). The point is that sometimes particular properties of the operator may be exploited to verify the appropriate testing criterion.

In particular, during the talk I would present some results for "square functions"with non-pointwise bounded kernels as well as the motivation that leads us to study such case. The work presented is a joint work with prof. Steve Hofmann.

Gressman, Philip (University of Pennsylvania, USA):
Scalar oscillatory integrals in smooth spaces of homogeneous type.

Abstract: We consider a generalization of the notion of spaces of homogeneous type, inspired by recent work of Street on the multi-parameter Carnot-Carathéodory geometry, which imbues such spaces with differentiability structure. The setting allows one to formulate estimates for scalar oscillatory integrals on these spaces which are uniform and respect the underlying geometry of both the space and the phase function. As a corollary we obtain a generalization of a theorem of Bruna, Nagel, and Wainger on the asymptotic behavior of scalar oscillatory integrals with smooth, convex phase of finite type.

Hagelstein, Paul (Baylor University, USA):
Transference of Weak Type Bounds of Multiparameter Ergodic and Geometric Maximal Operators.

Abstract: In this talk we will discuss recent work of Hagelstein and Stokolos regarding transference methods in multiparameter harmonic analysis and ergodic theory. Particular emphasis will be given on how these methods enable us to yield sharp weak type bounds on ergodic maximal operators associated to rare bases.

Hart, Jarod (University of Kansas):
Bilinear Square Functions and Vector-Valued Calderón-Zygmund Operators .

Abstract: Boundedness results for bilinear square functions and vector-valued operators on products of Lebesgue, Sobolev, and other spaces of smooth functions are presented. Almost orthogonality techniques are used to prove bilinear square function estimates for a range of Lebesgue space. Vector-valued bilinear Calderón-Zygmund operators are introduced and used to extend bounds for the optimal range of exponents of Lebesgue spaces, including some Lebesgue spaces with exponents smaller than one.

Hong, Sunggeum (Chosun University, Korea):
Remarks on the multiplier operators associated with a cylindrical distance function.

Abstract: In this note, we consider sharp L^p and maximal L^p estimates for the generalized Riesz means which are associated with the cylindrical distance function $\rho(\xi) = \text{máx}(|\xi'|, |\xi_{d+1}|)$, $\xi' \in \mathbb{R}^d$. We establish these estimates up to the currently known range of spherical Bochner-Riesz and its maximal operators. This is done by establishing implications between the corresponding estimates for the spherical Bochner-Riesz and the cylindrical multiplier operators. This is a collaboration with Sunghun Choi and Sanghyuk Lee.

Honzik, Petr (Charles University, Czech Republic):
Singular integrals with rough kernels

Abstract: For an integrable function Ω with mean value 0 defined on S^{n-1} we define the singular integral operator

$$T_\Omega f(x) = p.v. \int f(x-y)\frac{\Omega(y/|y|)}{|y|^n}dy.$$

We discuss the p dependent L^p boundedness of T_Ω and of the maximal version $T^*_\Omega f$.

Kairema, Anna (University of Helsinki, Finland):
On the adjacent systems of dyadic cubes in a metric space and applications to sharp weighted bounds.

Abstract: In this talk we discuss some recent progress on the constructions of dyadic cubes in a metric space, led by Michael Christ [1] and continued in [2]. The non-random construction of boundedly many adjacent families D^t of dyadic cubes, having the property that any ball B is contained in some $Q \in \bigcup_{t=1}^K \mathrm{D}^t$ with diam $(Q) \leq C$diam (B), is of particular interest here. It has been used to obtain extensions of Euclidean results on sharp weighted bounds for integral operators in spaces of homogeneous type.

As a first application of such adjacent dyadic families, we present a metric space version of Buckley's theorem on the sharp dependence of the operator norm on weight constant in Muckenhoupt's theorem for the Hardy–Littlewood maximal operator.

In the second application we consider a version of fractional integral operator on a space of homogeneous type. We propose a metric space version of the well-known Hardy–Littlewood–Sobolev theorem and find the sharp relationship between the operator norm on weighted spaces and the weight constant. Some of the main techniques used include a characterizations of two weight norm inequalities in terms of Sawyer-type testing conditions as well as a sharp version of the extrapolation theorem of Rubio de Francia. Our result generalizes the resent Euclidean result by Lacey, Moen, Pérez and Torres [3].

[1] Michael Christ. A $T(b)$ theorem with remarks on analytic capacity and the Cauchy integral. *Colloq. Math.*, 60/61:601–624, 1990.
[2] Tuomas Hytönen and Anna Kairema. Systems of dyadic cubes in a doubling metric space. *Colloq. Math.*, to appear.
[3] Michael T. Lacey, Kabe Moen, Carlos Pérez, and Rodolfo H. Torres. Sharp weighted bounds for fractional integral operators. *J. Funct. Anal.*, 259(5):1073–1097, 2010.

Kemppainen, Mikko (University of Helsinki, Finland):
On the atomic decomposition for the tent space T^1.

Abstract: In this talk I present an alternative proof of the atomic decomposition for the tent space T^1. The motivation for a new approach arose from the vector-valued setting [2], where the appearance of stochastic integrals rendered the original proof from [1] inapplicable. The new proof employs a 'cone covering lemma', which naturally applies to a description of atoms by averages of their square functions. Recently, the proof was found to apply also in the Gaussian setting, where an earlier approach relying on a Whitney covering argument was presented in [3]. The atomic decomposition for T^1 provides a way to derive molecular decompositions for Hardy spaces associated to more general elliptic operators than the Laplacian.

[1] R. R. Coifman, Y. Meyer, and E. M. Stein. Some new function spaces and their applications to harmonic analysis. *J. Funct. Anal.*, 62(2):304–335, 1985.
[2] M. Kemppainen. The vector-valued tent spaces T^1 and T^∞. *J. Aust. Math. Soc.* (to appear).
[3] J. Maas, J. van Neerven, and P. Portal. Whitney coverings and the tent spaces $T^{1,q}(\gamma)$ for the Gaussian measure. *Ark. Mat.* (to appear). Preprint, arXiv:1002.4911.

Kohr, Mirela (Babeş-Bolyai University, Cluj-Napoca, Romania):
Potential analysis for elliptic boundary value problems on Lipschitz domains in Riemannian manifolds.

Abstract: In this talk we present a potential analysis for some elliptic pseudodifferential matrix operators on Lipschitz domains in compact Riemannian manifolds. Applications will be given for boundary value problems related to Brinkman operators on such Lipschitz domains and boundary data in various Sobolev spaces.

Joint work with Cornel Pintea (Cluj-Napoca) and Wolfgang L. Wendland (Stuttgart).

Korobenko, Lyudmila (University of Calgary, Canada):
Regularity of solutions of degenerate quasilinear equations.

Abstract: In their paper of 1983 [1] C. Fefferman and D. H. Phong have characterized subellipticity of linear second order PDEs. The characterization is given in terms of subunit metric balls associated to the differential operator. An extension of this result to the case of the operator with non-smooth coefficients has been given by E. Sawyer and R. Wheeden in [2]. In this talk I am going to discuss the key points of their approach and an extension of the results to the case of non-doubling metric spaces. I will show how this technique can be used to prove continuity of weak solutions of infinitely degenerate quasilinear equations.

[1] C. Fefferman, D. H. Phong, Subelliptic eigenvalue problems, *Conf. in Honor of A. Zygmund*, Wadsworth Math. Series 1981.
[2] E. Sawyer, R. L. Wheeden, Hölder continuity of subelliptic equations with rough coefficients, *Mem. Amer. Math. Soc.* **847** (March, 2006).

Kovač, Vjekoslav (University of Zagreb, Croatia):
On Twisted Paraproducts and some other Multilinear Singular Integrals.

Abstract: Several years ago Demeter and Thiele posed a set of questions related to boundedness of certain multilinear singular integral operators. One of them was defined as

$T(f,g)(x,y) := \text{p.v.} \int_{\mathbb{R}^2} K(s,t)\, f(x-s,y)\, g(x,y-t)\, dsdt,$

where K is a (translation-invariant) 2D Calderón-Zygmund kernel. It reduces to a bilinear operator named *the twisted paraproduct*,

$\Pi(f,g)(x,y) := \sum_{k\in\mathbb{Z}} 2^{2k} \left(\int_{\mathbb{R}} f(x-s,y)\varphi(2^k s)ds\right) \left(\int_{\mathbb{R}} g(x,y-t)\psi(2^k t)dt\right),$

where φ, ψ are Schwartz and $\hat{\psi}$ is supported away from 0. Estimates for Π in a range of L^p spaces were shown recently, using dyadic operators and the Bellman function technique. We present this result and some more general results on multilinear operators established by the same method.

This is a joint work with Christoph Thiele, UCLA.

Liflyand, Elijah (Bar-Ilan University, Ramat-Gan, Israel):
Relations between the Fourier and Hilbert transforms.

Abstract: In this talk we discuss various conditions of the integrability of the Fourier transform of a function of bounded variation and their connections to the behavior of the Hilbert transform of a related function. Correspondingly, the considered spaces of functions with integrable Fourier transform are intimately related with the real Hardy space. One of the most important connections for the two transforms is given by the space introduced (for different purposes) by Johnson and Warner as $Q = \{g \in L^1(\mathbb{R}) \,:\, \int_{\mathbb{R}} \frac{|\widehat{g}(x)|}{|x|}\, dx < \infty\}$.

Lind, Martin (Karlstad University, Sweden):
On p-variation of functions of two variables.

Abstract: For functions of one variable, a generalization of the classical notion of the total variation (Jordan) was given in 1924 by Wiener, who in a similar way introduced the concept of the total p-variation of a function. Different extensions of the notion of generalized variation for functions of two variables have been studied by many authors. First, we consider one of them (p-variation of the Hardy-Vitali type) and obtain sharp estimates of the total p-variation in terms of the mixed modulus of continuity in $L^p([0,1]^2)$. We also investigate various embeddings for mixed norm spaces of functions whose linear sections have bounded p-variation in the sense of Wiener.

Martini, Alessio (Christian-Albrechts-Universität zu Kiel, Germany):
Sharp spectral multipliers for the Grushin operator.

Abstract: Let L be the Grushin operator acting on $\mathbb{R}^{d_1}_x \times \mathbb{R}^{d_2}_y$ and defined by the formula

$$L = -\sum_{j=1}^{d_1} \partial_{x_j}^2 - \left(\sum_{j=1}^{d_1} x_j^2 \right) \sum_{k=1}^{d_2} \partial_{y_k}^2 .$$

In a joint work with A. Sikora [`arXiv:1204.1159`], we obtain L^p spectral multiplier theorems and Bochner-Riesz summability for the Grushin operator L, which improve the known results, and are sharp in the case $d_1 \geq d_2$.

Namely, let $D = \max\{d_1 + d_2, 2d_2\}$, and let $W_2^s(\mathbb{R})$ denote the L^2 Sobolev space on $\mathbb{R}$ of (fractional) order s. Then we prove that an operator of the form $F(L)$ is of weak type $(1,1)$ and bounded on L^p for $1 < p < \infty$ whenever the multiplier function $F : \mathbb{R} \to \mathbb{C}$ satisfies $\sup_{t>0} \|F(t\cdot)\,\eta\|_{W_2^s} < \infty$ for some $s > D/2$ and some nonzero $\eta \in C_c^\infty(]0,\infty[)$. Correspondingly, we obtain that the Bochner-Riesz means $(1-tL)_+^\kappa$ are bounded on L^p for all $p \in [1,\infty]$, uniformly in $t > 0$, whenever $\kappa > (D-1)/2$.

Mas, Albert (Universidad del País Vasco – Euskal Herriko Unibertsitatea, Spain):
Variation for the Riesz transform and uniform rectifiability.

Abstract: I will present the following result, which is a joint work with Xavier Tolsa: *let $0 < n < d$ be integers and let μ be an n-dimensional AD regular measure in $\mathbb{R}^d$. Then, μ is uniformly n-rectifiable if and only if the variation for the Riesz transform with respect to μ is a bounded operator in* $L^2(\mu)$. This result is related to an important open problem, posed by David and Semmes, about the equivalence between uniform rectifiability and L^2 boundedness of the Riesz transform.

Mourgoglou, Mihalis (Université Paris-Sud 11, Orsay):
A local Tb theorem for square functions in domains with Ahlfors-David regular boundaries.

Abstract: We prove a "local" Tb Theorem for square functions, in which we assume L^p control of the pseudo-accretive system, with $p > 1$ extending the work of S. Hofmann to domains with Ahlfors-David regular boundaries.

Nilsson, Andreas (Saab AB, Sweden):
Invariant multipliers on $\mathbf{T}^n$.

Abstract: Multipliers correspond to translation invariant operators. Sometimes they satisfy more invariance conditions and this talk will be about such multipliers. For example Stein has shown that the Riesz transforms can be characterized as being invariant under dilations and satisfying a certain invariance condition under rotations. In this talk we consider the same kind of characterization for the Riesz transforms on $\mathbf{T}^n$ and give necessary and sufficient conditions for when this is possible.

Nowak, Adam (Polish Academy of Sciences, Poland):
Sharp estimates of the Jacobi heat kernel.

Abstract: The heat kernel associated with the setting of the classical Jacobi polynomials is defined by an oscillating sum which cannot be computed explicitly, in contrast to the situation for the two other classical systems of orthogonal polynomials named after Hermite and Laguerre. This sum is useless for estimating the kernel, and there is no better explicit formula. Only in 1960 it was shown that the kernel is positive.

We deduce sharp estimates giving the order of magnitude of the Jacobi heat kernel. We also give some applications of the bounds obtained. Joint work with Peter Sjögren, see `arXiv:1111.3145v1`.

Nyström, Kaj (Uppsala University, Sweden):
Sharp regularity for evolutionary obstacle problems, interpolative geometries and removable sets.

Abstract: In this paper we prove, by showing that solutions have exactly the same degree of regularity as the obstacle, optimal regularity results for obstacle problems involving evolutionary p-Laplace type operators. A technical key ingredient is the use of a new intrinsic interpolative geometry allowing for optimal linearization principles via blow-up analysis at contact points. This also opens the way to the proof of a removability theorem for solutions to evolutionary p-Laplace type equations. An important point of the paper is that, and this is in line with the corresponding linear results, no differentiability in time is assumed on the obstacle. This is joint work with Tuomo Kuusi and Giuseppe Mingione.

Oliveira e Silva, Diogo (University of California, Berkeley, United States):
On extremizers for Fourier restriction inequalities: the case of convex curves.

Abstract: This talk will focus on extremizers for a family of Fourier restriction inequalities on planar curves whose curvature satisfies a natural geometric assumption. It turns out that any extremizing sequence of nonnegative functions has a subsequence which converges to an extremizer. We hope to describe the method of proof, which is of concentration compactness flavor, in some detail. Tools include bilinear estimates, a variational calculation and a modification of the usual method of stationary phase.

Ott, Katharine (University of Kentucky, United States):
The mixed problem in Lipschitz domains.

Abstract: In this talk I will discuss the mixed problem, or Zaremba's problem, for second order elliptic operators in a bounded Lipschitz domain. Consider $\Omega \subset \mathbf{R}^n$, $n \geq 2$, a bounded Lipschitz domain with boundary $\partial\Omega$ decomposed $\partial\Omega = D \cup N$, with D and N disjoint. We specify Dirichlet boundary data on D and Neumann boundary data on the remainder of $\partial\Omega$. We seek conditions on the domain, the boundary, and the data which guarantee that the gradient of the solution of mixed problem lies in $L^p(\partial\Omega)$ for $1 \leq p < \infty$. I will present recent results of this nature when the underlying elliptic operator is the Laplacian or the Lamé system of elastostatics. This is joint work with Russell Brown and Justin Taylor.

Ou, Winston (Scripps College, United States):
Irregularity of Distributions and Multiparameter A_p Weights.

Abstract: Roth and Schmidt's seminal result in distribution theory, giving lower bounds on the L^p norm of the "discrepancy function"(measuring, for a collection of points in the unit cube, the discrepancy between the actual and expected number of points in a rectangle with one corner at the origin), is shown via weighted multiparameter Littlewood-Paley theory to be generalizable to the case where the measure is a product Muckenhoupt A_p weight.

Portal, Pierre (Australian National University, Australia):
Gaussian Hardy spaces.

Abstract: Gaussian harmonic analysis deals with the Ornstein-Uhlenbeck operator $Lf(x) = -\Delta f(x) + x.\nabla f(x)$ acting on function spaces such as $L^2(\mathbb{R}^n, \gamma)$, where γ denotes the gaussian measure. This is motivated by stochastics, where the semigroup $(e^{-tL})_{t\geq 0}$ is used as one of the most basic examples of a transition semigroup, with γ as invariant measure. From the point of view of harmonic analysis, this is particularly interesting because γ is non-doubling and e^{-tL} has a kernel that, in some sense, is far from being a standard Calderón-Zygmund kernel. It has been extensively studied, and specific techniques have been designed since the pioneering work of Muckenhoupt in the late 1960's.

In 2007, Mauceri and Meda introduced an atomic Hardy space $H^1_{at}(\mathbb{R}^n;\gamma)$ that provides a good end-point for interpolation, and is such that many operators associated with L are bounded from $H^1_{at}(\mathbb{R}^n;\gamma)$ to $L^1(\mathbb{R}^n;\gamma)$. However, in 2010, Mauceri, Meda and Sjögren realised that some Riesz transforms associated with L are bounded from $H^1_{at}(\mathbb{R}^n;\gamma)$ to $L^1(\mathbb{R}^n;\gamma)$ if and only if $n = 1$!

In this talk, we present an alternative Gaussian Hardy space $h^1(\mathbb{R}^n;\gamma)$ that can be defined through two equivalent norms - one involving a maximal function and one involving a square function - and that is such that the Riesz transforms are bounded from $h^1(\mathbb{R}^n;\gamma)$ to $L^1(\mathbb{R}^n;\gamma)$.

Reguera, Maria Carmen (University of Lund, Sweden):
Sharp Békollé estimates for the Bergman projection .

Abstract: Finding sharp estimates in terms of the Muckenhoupt A_p constant for singular integrals on weighted Lebesgue spaces has attracted a lot of attention lately. In the same spirit, we look for analogous estimates for the Bergman projection on the disc $\mathbb{D}$. The class of weights on $\mathbb{D}$ for which the Bergman projection P extends to a bounded linear operator on weighted spaces $L^p(w, \mathbb{D})$ is the so called Békollé-Bonami class and it is usually denoted by B_p. In this talk, we find the linear bound for the Bergman projection in terms of the B_2 constant. The proof will consider a novel dyadic model for the Bergman projection. This is joint work with Alexandru Aleman and Sandra Pott.

Rios, Cristian (University of Calgary, Canada):
The Kato problem for A_2-elliptic operators.

Abstract: We consider second order divergence form operators
$\mathcal{L}_w = -\frac{1}{w}\text{div}\mathbf{A}\nabla$ where w is an A_2 weight and the $n \times n$ complex-valued matrix $\mathbf{A}$ satisfies the weighted ellipticity and boundedness conditions

$$\lambda w|\xi|^2 \leq \text{Re}\,(\mathbf{A}\xi \cdot \xi^*) \quad \text{and} \quad |\mathbf{A}\xi \cdot \eta^*| \leq \Lambda w|\xi|\,|\eta|$$

for some $0 < \lambda \leq \Lambda < \infty$ and all $\xi, \eta \in \mathbf{C}^n$. We prove the Kato conjecture corresponding to this class of operators, namely:

$$||\mathcal{L}_w^{\frac{1}{2}} f||_{L^2(w)} \approx ||\nabla f||_{L^2(w)}$$

for all f in the domain of $\mathcal{L}_w$. This is work in collaboration with David Cruz-Uribe (Trinity College, USA).

Rodríguez-López, Salvador (Uppsala University , Sweden):
Global Boundedness of Multilinear Fourier Integral Operators.

Abstract:
In this talk we present some results about the global boundedness of multilinear Fourier integral operators, where the amplitudes are smooth or rough in the spatial variables.

We show some global boundedness results of rough linear Fourier integral operators with amplitudes that behave as L^p functions in the spatial variables, and as an amplitude in the Hörmander class $S^m_{\rho,0}$ in the frequency variable. We exploit these global linear results, in conjunction with an iteration procedure and a decomposition of the amplitudes, to study the boundedness for smooth or rough multilinear operators.

This is a joint work with Wolfgang Staubach (Uppsala University).

Roncal, Luz (Universidad de La Rioja, Spain):
The wave equation for the Bessel Laplacian.

Abstract: We consider the Cauchy problem for the wave equation in the multidimensional unit ball B^d, $d \geq 1$. When the inital data are radial, we are led to the equation

$$\frac{\partial^2}{\partial t^2} u(t,x) = -L_\nu u(t,x)$$

with initial conditions $u(0,x) = f(x)$, $\frac{\partial}{\partial t} u(0,x) = g(x)$, where

$$L_\nu = -\frac{d^2}{dx^2} - \frac{2\nu+1}{x}\frac{d}{dx}, \quad \nu > -1,$$

is the Bessel Laplacian. The solution $u(t,x)$ can be written as a Fourier-Bessel expansion. We prove that, for initial L^p data, such series is bounded in the L^2 norm. The analysis of the adjoint of the Riesz transform for Fourier-Bessel series is needed for our purposes, and it may be of independent interest. As application, certain $L^p - L^2$ estimates for the solution of the heat equation and the Cafarelli-Silvestre extension problem for the fractional Bessel Laplacian are obtained.

The work is in collaboration with professor Óscar Ciaurri (Universidad de La Rioja).

Rule, David (Heriot-Watt University, United Kingdom):
An end-point result for bilinear Fourier integral operators.

Abstract: I will describe an extension of a theorem of R. Coifman and Y. Meyer regarding bilinear pseudo-differential operators to bilinear Fourier integral operators. More precisely, we prove the global $L^2 \times L^2 \to L^1$ boundedness of bilinear Fourier integral operators with amplitudes in the class $S^0_{1,0}$. The proof makes use of a quadratic $T(1)$-theorem and commutator estimates. This is joint work with Wolfgang Staubach and Salvador Rodríguez-López.

Silvestre, Pilar (University of Barcelona, Spain):
Capacities and Embeddings via symmetrization and conductor inequalities.

Abstract: In this talk, using estimates of rearrangements in terms of modulus of continuity, some isocapacitary inequalities will be derived for Besov, Lipschitz or H^ω_p capacities. These inequalities will allow us to show that capacitary Lorentz spaces, based on Besov spaces, are between the homogeneous Besov spaces and the usual Lorentz spaces. Moreover, we will extend a result of Adams-Xiao to other function spaces and we will improve embeddings of Lipschitz and H^ω_p spaces in Lorentz spaces.

Slavin, Leonid (University of Cincinnati, USA):
Bellman functions for α-trees on non-convex domains.

Abstract: Inequalities for many important function classes, such as BMO and A_p, give rise to Bellman functions on non-convex domains. One often considers these classes in the dyadic setting or, more generally, on α-trees (any cube in $\mathbb{R}^n$ is the root of a dyadic 2^{-n}-tree). In such cases, the exact Bellman functions are difficult to compute, while substitutes typically produce crude dimensional behavior in the constants.

I will present a relatively general method of obtaining true Bellman functions – and thus proving sharp inequalities – in such settings. Two essential examples are provided by the dyadic maximal operator: $A_\infty \to A_1$ and the John–Nirenberg inequality for the dyadic BMO. Notably, the resulting sharp constants are exponential in dimension for BMO and saturated with dimension for the maximal function on A_∞.

This is joint work with Winston Ou and Vasily Vasyunin.

Stachura, Eric (Temple University, United States):
Mellin Transform Techniques for Singular Integral Operators.

Abstract: In this talk I will discuss spectral properties on the Lebesgue scale of p integrable functions of singular integral operators on polygons in two dimensions, $1 < p < \infty$. This analysis includes the physically relevant reflection operator, arising in the study of the radiosity equation on unoccluded surfaces, and the case of singular integral operators arising in connection with second and higher order elliptic boundary value problems (BVPs) in the plane. In the case of the reflection operator, we establish an explicit characterization of the spectrum on an infinite angle of aperture $\theta \in (0, \pi)$ and explicit spectral radius formulas which depend on the integrability index and on the angle of aperture.

Stinga, Pablo Raúl (Universidad de La Rioja, Spain):
Harnack's inequality for fractional Laplacians.

Abstract: We show Harnack's inequalities for fractional powers of *Laplacians* L^σ, $0 < \sigma < 1$. Our examples include second order divergence form elliptic operators with potentials, the radial Laplacian and operators arising in classical orthogonal expansions (Laplacian on the torus; Hermite, Laguerre and ultraspherical expansions; Bessel operators). The main idea is to apply a generalization of the Caffarelli-Silvestre extension problem for the fractional Laplacian. This generalization, that applies to any Laplacian L, is possible thanks to the introduction of heat-diffusion semigroups e^{-tL}. In this way we can take advantage of local methods, like Gutiérrez's Harnack inequality for degenerate Schrödinger equations, to obtain our results.

Stokolos, Alexander (Georgia Southern University, USA):
Monge-Ampère equations and Bellman functions: the dyadic maximal operator.

Abstract: When proving a sharp inequality in a harmonic analysis setting, one can sometimes recast the problem as that of finding the corresponding *Bellman function*. These functions often arise as solutions of Monge-Ampere PDEs on problem-specific domains; in such a case, the optimizers in the inequality can be found using the straight-line characteristics of the equation.

I will show how to find the Bellman function for one important example – the dyadic maximal operator on L^p. This function has been previously found by A. Melas in a different way. The approach presented can be generalized to other well-localized operators and function classes. Joint work with Leonid Slavin and Vasily Vasyunin.

Tikhonov, Sergey (ICREA and CRM, Spain):
Sharp local inequalities for trigonometric polynomials.

Abstract: We discuss a sharp Remez inequality for the trigonometric polynomials and its applications.

Tsutsui, Yohei (Waseda University, Japan):
A_∞ constants between BMO and weighted BMO.

Abstract: It was proved by B. Muckenhoupt and R.L. Wheeden in 1976 that for every weight w belonging to Muckenhoutp class A_∞, $BMO = BMO(w)$, where

$$\|f\|_{BMO(w)} = \sup_Q \frac{1}{w(Q)} \int_Q |f - \langle f\rangle_{Q;w}| w dx,$$

and $w(Q) = \int_Q w dx$, $\langle f\rangle_{Q;w} = \frac{1}{w(Q)} \int_Q f w dx$. In this talk, we consider estimates of the ratio $\frac{\|f\|_{BMO(w)}}{\|f\|_{BMO}}$ from above and below in terms of the A_∞ constant of the weight w.

Urbina, Wilfredo (Roosevelt University Chicago, USA)
Transference results from the L^p continuity of operators in the Jacobi case to the L^p continuity of operators in the Hermite and Laguerre case.

Abstract: Using the well known asymptotic relations between Jacobi polynomials and Hermite and Laguerre polynomials we develop a transference method to obtain the L^p-continuity of the Gaussian-Riesz transform and the L^p-continuity of the Laguerre-Riesz transform from the L^p-continuity of the Jacobi-Riesz transform, in dimension one as well as the L^p-continuity of the Gaussian-Riesz transform and the L^p-continuity of the Laguerre-Riesz transform from the L^p-continuity of the Jacobi-Riesz transform. Joint work with Eduard Navas (Universidad Experimental Francisco de Miranda, Punto Fijo, Venezuela).

Uriarte-Tuero, Ignacio (Michigan State University, USA):
On the two weight problem for the Hilbert transform.

Abstract: Let σ and ω be locally finite positive Borel measures on $\mathbb{R}$ which do not share a common point mass. Then, the Hilbert transform $H(\sigma f)$ maps from $L^2(\sigma)$ to $L^2(\omega)$ if and only if $H(\sigma f)$ maps $L^2(\sigma)$ into weak-$L^2(\omega)$, and the dual weak-type inequality holds. This is a corollary to a more precise characterization in terms of a Poisson A_2 condition on the pair of weights, and conditions phrased in terms of testing the norm inequality over bounded functions supported on an arbitrary interval.

Joint work with M. Lacey, E. Sawyer and C.-Y. Shen.

Valdimarsson, Stefán Ingi (University of Iceland, Iceland):
Multilinear Inequalities and Functional Analysis.

Abstract: In this talk we discuss a duality principle for certain multilinear inequalities. This is inspired by Guth's proof of the Multilinear Kakeya Inequality where the proof of the inequality is reduced to showing that a convex optimization problem has a solution. We show that the converse is true, that is, the inequality implies the existence of a solution to the optimization problem. Moreover, we demonstrate that this is a general functional analytic principle and give examples of its application.

This is joint work with A. Carbery.

Villarroya Alvarez, Paco (Lunds Universitet, Sweden):
$T(1)$ Theory for compact singular integral operators.

Abstract: We prove a $T(1)$-type theorem which characterizes compactness of singular integral operators whose kernels satisfy a smoothness condition with decay along the diagonal.

In the same spirit of David and Journé's original $T(1)$ Theorem, we provide this characterization in terms of the action of the operator over special families of functions. For this reason, we define a new property of 'weak compactness' and a proper substitute for the space BMO.

We apply our main theorem to prove compactness of some particular perturbations of the Cauchy Integral.

Wróbel, Błażej (University of Wrocław, Poland):
Multivariate spectral multipliers.

Abstract: Using a multi-dimensional analogue of the Mellin transform techniques we prove a multivariate multiplier theorem for general tensor product orthogonal expansions. For operators of the form

$$\mathcal{L}_n f = \sum_{r=0}^{\infty} \lambda_r^n \langle f, e_r^n \rangle e_r^n, \qquad n = 1, \dots, d,$$

their joint spectral multiplier is defined as

$$m(\mathcal{L})f = \sum_{k \in \mathbb{N}^d} m(\lambda_{k_1}^1, \dots, \lambda_{k_d}^d) \langle f, e_k \rangle e_k, \quad e_k = e_{k_1} \otimes \cdots \otimes e_{k_d}.$$

We give conditions on the function m under which $m(\mathcal{L})$ is bounded on L^p spaces, $1 < p < \infty$. In the case of polynomially growing norms of the imaginary powers of $\mathcal{L}_n$, to get the L^p boundedness of $m(\mathcal{L})$ it suffices to assume a specific product Marcinkiewicz type condition. We also obtain a multivariate multiplier theorem for the Hankel transform.

Zhang, Chao (Wuhan University, China, and Universidad Autónoma de Madrid, Spain):
Regularity properties of Schrödinger operators.

Abstract: In this talk we use a method of $\mathcal{L}$-harmonic extensions to study the regularity estimates at the scale of adapted Hölder spaces, where $\mathcal{L}$ is a Schrödinger operator of the form $\mathcal{L} = \Delta + V$ and the nonnegative potential V satisfies a reverse Hölder inequality. Parallel to the estimates obtained by Silvestre for the classical fractional laplacian, we get regularity estimates for the positive powers of $\mathcal{L}$. We give a pointwise description of the $\mathcal{L}$-Hölder spaces and provide some characterizations in terms of the growth of fractional derivatives and Carleson measures. Applications to Riesz potentials and Laplace transform type multipliers are also developed.

Zubeldia, Miren (University of the Basque Country, Spain):
The forward problem for the electromagnetic Helmholtz equation.

Abstract: In this talk we show recent results related to the uniform resolvent estimates and Sommerfeld radiation condition for solutions $u \in H_A^1(\mathbb{R}^d)$ $(d \geq 3)$ of the electromagnetic Helmholtz equation

$$(\nabla + iA(x))^2 u + V(x)u + \lambda u + i\varepsilon u = f$$

with singular potentials. We then deduce the limiting absorption principle and some spectral properties of the magnetic Schrödinger operator. We use a multiplier method and integration by parts as a main tools.