Aspects of harmonic analysis related to hypersurfaces and Newton diagrams

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Abstract

For several decades now, a major part of research in Euclidean harmonic analysis is driven by the problem to understand the interplay between geometric properties, such as curvature properties of certain subvarieties, and related operators, such as maximal averaging operators along hypersurafces, Bochner-Riesz-multipliers, various classes of oscillatory integral operators, or (singular) Radon transforms, to give just a few examples. Important instances are Stein's spherical maximal operator, or Fourier restriction operators to spheres or cones. Naturally, a lot of attention has been given to the most generic situations, for instance hypersurfaces with non-vanishing curvature, for which deep, but in general still not complete results have been obtained.

However, what can be said about more general varieties, with weaker geometric properties? For some of these problems, related to smooth, finite type hypersurfaces S in $\mathbb{R}^3$, in joint work with I. Ikromov we have been able to give essentially complete answers in terms of Newton diagrams associated to the given hypersurface. More precisely, if we denote by $d\mu = \rho d\sigma$ a surface carried measure with smooth, compactly supported density $\rho \geq 0$ with respect to the surface measure $d\sigma$, then I shall discuss in particular the following questions:

A. Find the best possible uniform decay estimates for the Fourier transform $\hat{\mu}$ of the surface carried measure $d\mu$.

B. Determine the range of exponents p for which a Fourier restriction estimate

$$\Big(\int_S |\hat{f}(x)|^2 \, d\mu(x)\Big)^{1/2} \leq C\|f\|_{L^p(\mathbb{R}^3)}$$

holds true. The first problem is classical, and the second problem has been introduced by E.M. Stein. Indeed, owing a lot to the insight of Arnol'd and his school, it is known that the first question can be

answered by means of Newton diagrams when S is analytic, and we have shown that it is possible to extend this to the non-analytic case.

The program of my talks will thus comprise a review of various important notions, such as Newton polyhedra, adaptedness of a given coordinate system, etc.. Subsequently I shall try to give an idea how properties of the Newton polyhedra attached to the surface will reflect the decay rate of $\hat{\mu}(\xi)$, and finally I shall address question B and indicate some major ideas used in the proof of our main results. We shall see in particular that problems A and B are not as directly related as one might perhaps expect.