An H_1-BMO duality for semigroups of operators

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Abstract

Let (T_t) be an abstract Markov diffusion semigroup of operators on a measure space. We may define a BMO norm associated with (T_t) as $\|f\|_{BMO} = \sup_t \|T_t|f - T_t f|^2\|_\infty^{\frac{1}{2}}$, viewing T_t as an alternative of the mean value operator. The question is how to define an corresponding H_1 norm and to establish Fefferman-Stein's duality theory in this abstract setting, and what conditions on (T_t) are needed? We assume there is no direct information available on the local structure of the underlying measure space and seek for a duality theory relies merely on the semigroups of operators.

A main motivation of this question is from our recent work on noncommutative Fourier multipliers, jointly with Junge and Parcet, where we have extensively used the abstract BMO norm given above in developing a Calderón-Zygmund theory on noncommutative L_p spaces. A main difficulty in the research is to find the "right" noncommutative alternatives to the geometric properties/tools used in the classical analysis.

In this talk, I will limit myself in the setting described in the first paragraph and report recent progress in seeking the desired duality theory by considering H_1 norms defined by semigroup-analogues of Lusin area integrals.