Pointwise convergence of vector-valued Fourier series

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Abstract

I report on recent joint work with M. Lacey, where we obtain a vector-valued version of Carleson's theorem: Let Y be a complex interpolation space between a UMD space X and a Hilbert space H. Then the Fourier series of Y-valued functions converge pointwise almost everywhere.

Apparently, all known examples of UMD spaces are intermediate spaces as described. In particular, this covers the noncommutative L^p spaces, and hence we answer affirmatively a question raised by J. L. Rubio de Francia in the 1980's on the convergence of Fourier series of Schatten class valued functions.

The proof consists of extending the Lacey–Thiele approach to time-frequency analysis to the vector-valued setting. We have also considered other related questions in this set-up, including the convergence of vector-valued Walsh series and, jointly with I. Parissis, a vector-valued Walsh model for the bilinear Hilbert transform.