# Performance of greedy algorithm in reduced basis method

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**Abstract**

The reduced basis method was introduced for the accurate online evaluation of solutions to a parameter dependent family of elliptic partial differential equations. Abstractly, it can be viewed as determining a "good" $n$ dimensional space $\mathcal{H}_n$ to be used in approximating the elements of a compact set $\mathcal{F}$ in a Hilbert or Banach space $\mathcal{H}$ where solutions live. One, by now popular, computational approach is to find $\mathcal{H}_n$ through a greedy strategy. It is natural to compare the approximation performance of the $\mathcal{H}_n$ generated by this strategy with that of the Kolmogorov widths $d_n(\mathcal{F})$ since the latter gives the smallest error that can be achieved by subspaces of fixed dimension $n$. The first such comparisons, given in A. Buffa, Y. Maday, A.T. Patera, C. Prud'homme, and G. Turinici, *A Priori convergence of the greedy algorithm for the parameterized reduced basis* M2AN Math. Model. Numer. Anal., **46**(2012), 595–603, show that the approximation error in a Hilbert space, $\sigma_n(\mathcal{F}) := \operatorname{dist}(\mathcal{F}, \mathcal{H}_n)$, obtained by the greedy strategy satisfies $\sigma_n(\mathcal{F}) \leq Cn2^n d_n(\mathcal{F})$. In this talk, various improvements of this result will be given both in Hilbert and in Banach space case. We discuss both individual comparison between $\sigma_n(\mathcal{F})$ and $d_s(\mathcal{F})$ and the estimates for classes when we assume certain decay of $d_n(\mathcal{F})$ and obtain related decay of $\sigma_n(\mathcal{F})$.

This talk reports the joint work with Peter Binev, Albert Cohen, Wolfgang Dahmen, Ronald DeVore and Guergana Petrova.