## Invited Addresses

**Akram Aldroubi** (Vanderbilt University)
*Dynamical Sampling and Systems of Iterative Action of Operators.*

**Abstract:** We consider frames and Bessel systems generated by iterations of the form $\{A^n g : g \in \mathcal{G}, 0 \leq n < L(g)\}$, where $A$ is a bounded linear operators on a separable complex Hilbert space $\mathbb{H}$ and $\mathcal{G}$ is a countable set of vectors in $\mathbb{H}$. The system of iterations mentioned above come from the so called *dynamical sampling problem.* In dynamical sampling, an unknown function $f$ and its future states $A^n f$ are coarsely sampled at each time level $n$, $0 \leq n < L$, where $A$ is an evolution operator that drives the system. The goal is to recover $f$ from these space-time samples. The dynamical sampling problem has connections and applications to other areas of mathematics including, Banach algebras, $C^*$-algebras, spectral theory of normal operators, and frame theory.

**Nets Katz** (Caltech)
*Scenes from Kakeya sets in $\mathbb{R}^3$ near the Wolff exponent.*

**Abstract** (joint work with Josh Zahl): We discuss some vignettes from an attempt to improve Wolff's lower bound in $\mathbb{R}^3$. Conditional on a strong result of discretized sum product type, only a few different scenarios need be considered.

**Andrei Lerner** (Bar-Ilan University)
*On pointwise estimates involving sparse operators.*

**Abstract:** In this talk we survey several recent results establishing a pointwise domination of Calderón-Zygmund operators by sparse operators defined by

$$\mathcal{A}_{\mathcal{S}} f(x) = \sum_{Q \in \mathcal{S}} \Big( \frac{1}{|Q|} \int_Q f \Big) \chi_Q(x),$$

where $\mathcal{S}$ is a sparse family of cubes from $\mathbb{R}^n$.

We will also discuss an analogue of such a domination for commutators of Calderón-Zygmund operators, obtained in a joint work with S. Ombrosi and I. Rivera-Ríos.

**Svitlana Mayboroda** (University of Minnesota)
*Harmonic measure in higher co-dimension.*

**Abstract:** Harmonic measure and harmonic functions more generally play a unique role, in particular, in geometric measure theory: boundedness of the harmonic Riesz transform is equivalent to uniform rectifiability of sets, so is the boundedness of the harmonic square function, to mention only a few results. Unfortunately, the concept of the harmonic measure is intrinsically restricted to co-dimension 1. In this talk, we introduce a new notion of a "harmonic" measure, also associated to a PDE, which serves the higher co-dimensions. We discuss its basic properties and give big strokes of the argument to prove that our measure is $A^\infty$ on Lipschitz graphs with small Lipschitz constant.

**Ursula Molter** (University of Buenos Aires and IMAS - CONICET)
*The Amalgan Balian Low Theorem and time-frequency shift invariance.*
**Abstract:** The *Balian-Low Theorem* expresses the fact that time-frequency concentration and non redundancy are essentially incompatible. Specifically, if $\varphi \in L^2(\mathbb{R})$, $\Lambda \subset \mathbb{R}^2$ is a lattice and the system $(\varphi, \Lambda) = \{e^{2\pi i \eta x}\varphi(x-u) : (u, \eta) \in \Lambda\}$ is a Riesz basis for $L^2(\mathbb{R})$, then $\varphi$ satisfies

$$\Big(\int (x-a)^2|\varphi(x)|^2\,dx\Big)\cdot\Big(\int(\omega-b)^2|\widehat{\varphi}(\omega)|^2\,d\omega\Big) = \infty, \quad a, b \in \mathbb{R}.$$

The *Amalgam Balian-Low Theorem* states that if $(\varphi, \alpha\mathbb{Z} \times \beta\mathbb{Z})$ is a Riesz basis for $L^2(\mathbb{R})$, then $\varphi$ cannot belong to the Feichtinger algebra $S_0(\mathbb{R})$, a class of functions decaying well in time and frequency. Precisely,

$$S_0(\mathbb{R}) = \left\{ f \in L^2(\mathbb{R}) : Vf(t,\nu) = \int f(x)e^{-(x-t)^2}e^{2\pi i x\nu}\,dx \in L^1(t,\nu)\right\}.$$

Note that $Vf(t, \nu) \in L^2(t, \nu) \cap L^\infty(t, \nu)$ for all $f \in L^2(\mathbb{R})$ and the requirement $Vf(t, \nu) \in L^1(t, \nu)$ essentially necessitates $L^1$ decay of $f$ and of its Fourier transform $\widehat{f}$. This space is called the *Feichtinger algebra*.

Let $T_u f(x) = f(x-u)$, and $M_\eta f(x) = e^{2\pi i \eta x} f(x)$, denote the usual translation and modulation operators, and let $\pi(u, \eta) = M_\eta T_u$, (with $u \in \mathbb{R}$ and $\eta \in \widehat{\mathbb{R}}$ the dual group of $\mathbb{R}$) denote the time-frequency shift. For $\varphi \in L^2(\mathbb{R})$ and a lattice $\Lambda \subset \mathbb{R} \times \widehat{\mathbb{R}}$, let $\mathcal{G}(\varphi, \Lambda)$ denote the *Gabor spaces*, $\mathcal{G}(\varphi, \Lambda) := \overline{\text{span}\,\{\pi(\lambda)\varphi\}}$, where $\overline{V}$ is the closure of $V$ in $L^2(\mathbb{R})$.

In this talk we address the question whether there may exist a $\mu \in \mathbb{R} \times \widehat{\mathbb{R}} \setminus \Lambda$ with $\pi(\mu)\varphi \in \mathcal{G}(\varphi, \Lambda)$. The result relates the existence of such $\mu$, to the fact that $\varphi$ belongs (or does not belong) to the *smoothness space* $S_0(\mathbb{R})$. We have

**Theorem.** *If $(\varphi, \Lambda)$ is a Riesz basis for its closed linear span $\mathcal{G}(\varphi, \Lambda)$ with $\varphi \in S_0(\mathbb{R})$ and the density of the lattice $\Lambda$ is rational, then for any $(u, \eta) \notin \Lambda$ $\pi(u, \eta)\varphi \notin \mathcal{G}(\varphi, \Lambda)$ .*

Note that $(\varphi, \Lambda)$ being a Riesz basis for $L^2(\mathbb{R})$ implies that the density of $\Lambda$ equals $1 \in \mathbb{Q}$; and $\mathcal{G}(\varphi, \Lambda) = L^2(\mathbb{R})$ implies that $\pi(u, \eta)\varphi \in \mathcal{G}(\varphi, \Lambda)$ for all $(u, \eta) \in \mathbb{R} \times \widehat{\mathbb{R}}$. Therefore the theorem implies that $\varphi \notin S_0(\mathbb{R})$.

Joint work with: Carlos Cabrelli, University of Buenos Aires and IMAS-CONICET, Argentina; Götz Pfander, Philipps-Universität Marburg, Germany; and Dae Gwan Lee, Philipps-Universität Marburg, Germany.

**Alexander Olevskii** (Tel Aviv University.)
*Fourier quasicrystals.*

**Abstract:** By a Fourier quasicrystal one often means a discrete measure in $R^n$ with a pure point spectrum. A classical example of such a measure is given by the Poisson summation formula. Meyer's "model sets"provide a family of examples of discrete measures with a dense spectrum. A new peak of interest to the subject appeared in the middle 80's after the experimental discovery of physical quasicrystals.

I'll discuss the background and present our joint results with Nir Lev on the periodicity conjecture for discrete quasicrystals.

**Eric Ricard** (Université de Caen Normandie)
*Noncommutative De Leeuw's theorems.*

**Abstract:** A classical compactification theorem by De Leeuw asserts that a continuous function on $\mathbb{R}$ is a Fourier multiplier on $L_p(\mathbb{R})$ iff it is also a Fourier multiplier on $L_p(\mathbb{R}_d)$ where $\mathbb{R}_d$ is $\mathbb{R}$ with the discrete topology. It has been extended to all locally abelian amenable groups by Saeki using structure theory. This result can be very useful when one wants to transfer results about multipliers from one group to another. Motivated by the recent developments of the theory of $Lp$-multipliers in noncommutative analysis, one could look for suitable generalizations. A more functional analytic approach has to be developed. This is a joint work with M. Caspers, J. Parcet and M. Perrin.

**Mikko Salo** (University of Jyväskylä)
*Recent progress in the Calderon problem.*

**Abstract:** The inverse conductivity problem, posed by A.P. Calderon in 1980, consists in determining the coefficient $A$ in the elliptic PDE $div(A\nabla u) = 0$ from the Cauchy data of its solutions. This problem is the mathematical model for Electrical Impedance Tomography. Various harmonic analysis, PDE and geometric techniques come into play in its study, and the Calderon problem remains a central question in the theory of inverse problems. We will survey known results and open questions, focusing on issues with low regularity, partial data and matrix coefficients.

**Christoph Thiele** (University of Bonn)
*The harmonic analysis of two commuting transformations.*

**Abstract:** A transference principle allows to relate questions in ergodic theory with questions in harmonic analysis. Following this principle, convergence questions for bilinear ergodic averages with respect to two commuting transformations have lead to a particular program of study in harmonic analysis which has seen steady progress in recent years. We discuss several results including some joint work with P. Durcik, V. Kovac, and K. Skreb: On optimal quantitative norm convergence of ergodic averages for two commuting transformations.

**Xavier Tolsa** (ICREA / Universitat Autònoma de Barcelona)
*The Riesz transform, quantitative rectifiabilty, and a two-phase problem for harmonic measure.*

**Abstract:** A remarkable theorem of Léger asserts that if $\mu$ is a Radon measure in the Euclidean space and $B$ is a ball such that $\mu(B) = r(B)$ (where $r(B)$ stands for the radius of $B$) satisfying the linear growth condition $\mu(B(x,r)) \leq Cr$ for all $x, r$, and so that the curvature of $\mu$

$$c^2(\mu) = \iiint \frac{1}{R(x,y,z)^2}\, d\mu(x)d\mu(y)d\mu(z)$$

is small enough, then a big piece of $\mu$ on $B$ is supported on a Lipschitz graph and is absolutely continuous with respect to arc length measure on the graph. In the first part of my talk I will present a version of this theorem by Girela-Sarrión and myself which involves the $L^2$ norm of the codimension 1 Riesz transform in the Euclidean space.

In the second part I will explain an application (by Azzam, Mourgoglou and myself) of the preceding result to solve a two-phase problem for harmonic measure posed by Chris Bishop in 1990. This asserts that, for disjoint domains in the Euclidean space whose boundaries satisfy a non-degeneracy condition, mutual absolute continuity of their harmonic measures implies absolute continuity with respect to surface measure and rectifiability in the intersection of their boundaries. Up to now, by a result of Kenig and Toro, it was only known that the preceding condition implies that the harmonic measure is concentrated in a set of Hausdorff codimension 1.

**Jim Wright** (University of Edinburgh)
*On a theorem of L. Alpár.*

**Abstract:** In the 1950's, Leibenzon and Kahane showed that the only change of variables $w(t)$ of the circle group $\mathbb{T}$ which preserve the space of absolutely convergent Fourier series $A(\mathbb{T})$ (that is, $f(t) \in A(\mathbb{T})$ implies $f(w(t)) \in A(\mathbb{T})$) are affine ones $w(t) = at + b$. Kahane then went on to ask what happens when the target space $A(\mathbb{T})$ is replaced by the larger space of uniformly convergent Fourier series $U(\mathbb{T})$. A key result in this line of investigation is a result of Alpár who showed that any real-analytic function w works. Furthermore, one cannot weaken the change of variable to smooth $w$. We will investigate higher dimensional versions of Alpár's theorem.