Courses

Pascal Auscher (Université de Paris-Sud, CNRS)
The first order approach for elliptic and parabolic equations.

Abstract: The first order approach to elliptic equations is an old concept, going back to the relation between the Laplace equation (harmonic functions) and the Cauchy-Riemann system (holomorphic functions) in two dimensions. The higher dimensional version uses conjugate systems and this is how Stein and Weiss developed in 1960 the theory of (real) Hardy spaces on Euclidean spaces that became so popular and useful. For non-smooth elliptic equations in any dimension, this relation has been explored only in the last few years. While it builds on some simple algebra, the needed L^2 estimates rely on a more recent tool: more precisely, on one version of the T(b) theorem elaborated after the solution of the Kato problem solved fifteen years ago. Extrapolation and interpolation allows to obtain a theory in some other functional spaces. For example, it enables to show that for a large class of equations, solutions of boundary value problems of Dirichlet and Neumann type, provided they exist, must be represented by layer potentials. It also recently appeared that there is a first order approach to parabolic equations. In these lectures, I will present some aspects of the first order approach which has involved a number of authors and/or collaborators: A. McIntosh, A. Nahmod, A. Rosén, S. Keith, S. Hofmann, S. Stahlhut, M. Mourgoglou, Y. Huang, P. Portal, D. Frey, A. Amenta, M. Egert, K. Nyström.

Ciprian Demeter (Indiana University)
Decouplings and applications.

Abstract: I will describe how the decoupling methods developed jointly with Jean Bourgain and most recently with Larry Guth, produce sharp estimates for a wide variety of problems in PDEs and number theory.

Camillo de Lellis (Universität Zürich)
Surprising solutions to the isentropic Euler system of gas dynamics.

Abstract: In a recent paper, jointly with Elisabetta Chiodaroli and Ondřej Kreml we consider the Cauchy problem for the isentropic compressible Euler system in 22 space dimensions, with initial data which assume two different constant values and have a discontinuity across a line. If we consider selfsimilar solutions we then encounter a classical 11-dimensional Riemann problem for the corresponding hyperbolic system of conservation laws. We show that for some suitable choice of the pressure and of the initial data there exist infinitely many bounded admissible solutions which are not selfsimilar and indeed are genuinely 22-dimensional. We also show that some of these Riemann data are generated by a 11-dimensional compression wave. Our theorem leads therefore to Lipschitz initial data for which there are infinitely many global bounded admissible weak solutions. Each of these solutions coincide as long as the classical (Lipschitz) solution exists and they differentiate themselves immediately after the first blow-up time. In a further refinement Chiodaroli and Kreml have also shown that some of the non selfsimilar solutions dissipate more energy than the classical one.

Our approach is heavily influenced by a work of László Székelyhidi which provides a similar result in the case of the classical vortex-sheet problem for the incompressible Euler equations.

Marius Junge (Univeristy of Illinois)
Harmonic analysis for noncommutative spaces.
Talk I: *Matrix-valued singular integral theory and functional calculus.*
Abstract: In this talk we present a recent result by Sukochev and his coauthors proving a weak-type $(1,1)$ inequality for functional calculus obtained from applying a Lipschitz function to a selfadjoint matrix. The key ingredient is Parcet's result for matrix-valued singular integrals.

Talk II: *Riesz transform on discrete groups.*
Abstract: Using the algebraic setup of coycles on discrete groups we show how to obtain estimates for Fourier multipliers from noncommutative Riesz transforms. As an application we show that on $\mathbb{R}^n$ Hörmander Michlin multipliers are Littlewood Paley averages of Riesz transforms.

Talk III: *Harmonic analysis on noncommutative Euclidean spaces.*
Abstract: In some part of physics literature the instanton algebra is a deformed version of $\mathbb{R}^4$. We will show that noncommutative (or quantum) version of euclidean planes behave in many ways similar in terms of harmonic analysis as the commutative counterparts. In particular, we find will find heat kernel and dispersive estimates for these objects and propose a notion of singularity on 'pointless spaces'.

These presentations are based on joint work with Parcet, Mei and Zhao.