SEMINARIO DE ANÁLISIS Y APLICACIONES

Martes, 29 de octubre de 2019

15:00 h., Módulo 17 - Aula 520 (Depto. Matemáticas UAM)

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Fourier uncertainty principles, interpolation and uniqueness sets.

Resumen:

A classical result in the theory of entire functions of exponential type, Shannon's interpolation formula predicates that, given a function whose Fourier transform vanishes outside the interval $[-1/2, 1/2]$, it is possible to recover it from its values at the integers. More specifically, it holds, in a suitable sense of convergence, that

$$f(x) = \sum_{n\in\mathbb{Z}} f(n)\frac{\sin(\pi(x-n))}{\pi(x-n)}.$$

This formula is unfortunately unavailable for arbitrary Schwartz functions on the real line, but a recent result of Radchenko and Viazovska provides us with an explicit construction of an interpolation basis for even Schwartz functions. It states, in a nutshell, that we can recover explicitly the function given its values at the square roots of integers.

We will discuss a bit these results and explore the question of determining which pairs of sets (A, B) satisfy that, if a Schwartz function f vanishes on A and its Fourier transform vanishes on B, then $f \equiv 0$, with particular interest in the cases where $A = \{\pm n^\alpha\}_{n\in\mathbb{N}}$ and $B = \{\pm n^\beta\}_{n\in\mathbb{N}}$ are sets of powers of integers.