Seminario de Análisis y Aplicaciones

Viernes, 17 de mayo de 2019

11:30 h., Aula Gris 1 (ICMAT)

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Hardy spaces for Fourier integral operators

Resumen:

Which subspaces $X_p \subset L^p(\mathbb{R}^d)$ give good initial data for the wave equation $\partial_t^2 u = \Delta u$, in the sense that $u(0,.) \in X_p, \partial_t u(0,.) = 0$ implies $u(t,.) \in L^p$? A classical answer is $X_p = W^{(d-1)|1/p-1/2|,p}$. Its weakness is that, while $u(t,.) \in L^p$, one does not have $u(t,.) \in X_p$, i.e. one looses derivatives. In this talk, we consider a new Hardy space that contains $W^{(d-1)|1/p-1/2|,p}$, but is invariant under the action of the wave group. More generally, we show that this space is invariant under the action of a large class of Fourier Integral Operators. This allows us to recover and extend (to symbols that are less regular, and not compactly supported in space) a celebrated result of Seeger-Sogge-Stein on the L^p boundedness of FIO. More generally we set up a Hardy space theory that is meant to do for hyperbolic equations what the usual Hardy space theory does for elliptic and parabolic equations. This is possible because FIOs lifted to function spaces over phase space through appropriate wave packet transforms turn out to have a diffusive behaviour (not in space but in phase space, and with respect to an appropriate metric). In this talk, we discuss this perspective, the deep ideas from microlocal analysis and harmonic analysis that it builds upon, and its potential.

This is joint work with Andrew Hassell and Jan Rozendaal (ANU).