SEMINARIO DE ANÁLISIS Y APLICACIONES

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11:30 h., Módulo 17 - Aula 520 (Depto. Matemáticas UAM)

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Regularity theory and Green functions for elliptic equations with lower order terms in unbounded domains

Resumen:

In this talk we will discuss the extension of the regularity theory for solutions of elliptic PDE in divergence form $Lu = -\mathrm{div}(A \cdot \nabla u)$ with merely bounded coefficients in unbounded domains to operators of the form $Lu = -\mathrm{div}(A \cdot \nabla u + bu) - c \cdot \nabla u - du$, in an open set $\Omega \subset \mathbb{R}^n$, $n \geq 3$, with possibly infinite Lebesgue measure. We assume that the $n \times n$ matrix A is uniformly elliptic with real, merely bounded and possibly non-symmetric coefficients, and either $b, c \in L^{n;\infty}_{\mathrm{loc}}(\Omega)$ and $d \in L^{\frac{n}{2};\infty}_{\mathrm{loc}}(\Omega)$, or $|b|^2, |c|^2, |d| \in \mathcal{K}_{\mathrm{loc}}(\Omega)$, where $\mathcal{K}_{\mathrm{loc}}(\Omega)$ stands for the local Stummel-Kato class. Let $\mathcal{K}_{\mathrm{Dini},2}(\Omega)$ be a variant of $\mathcal{K}(\Omega)$ satisfying a Carleson-Dini-type condition. We develop a De Giorgi/Nash/Moser theory and also prove a Wiener-type criterion for boundary regularity. Assuming global conditions on the coefficients, we show that the variational Dirichlet problem is well-posed and, assuming $-\mathrm{div}c + d \leq 0$, we construct the Green's function associated with L satisfying quantitative estimates. Under the additional hypothesis $|b + c|^2 \in \mathcal{K}'(\Omega)$, we show that it satisfies global pointwise bounds and also construct the Green's function associated with the formal adjoint operator of L. An important feature of our results is that all the estimates are scale invariant and independent of Ω, while we do not assume smallness of the norms of the coefficients or coercivity of the associated bilinear form.