Seminario de Análisis y Aplicaciones

Viernes, 29 de noviembre de 2019

10:30 h., Aula Gris1 (ICMAT)

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The Dual of variable Lebesgue spaces with unbounded exponent

Resumen:

Given a measure space (X, μ) and a measurable function $p(\cdot) : X \to [1, \infty]$, the variable Lebesgue space $L^{p(\cdot)}(X)$ consists of all measurable functions f such that for some $\lambda > 0$,

$$\rho(f/\lambda) = \int_{X_*} \left(\frac{|f(x)|}{\lambda}\right)^{p(x)} d\mu + \lambda^{-1}\|f\|_{L^\infty(X_\infty)},$$

where $X_* = \{x \in X : p(x) < \infty\}$ and $X_\infty = \{x \in X : p(x) = \infty\}$. This space is a Banach function space with the norm

$$\|f\|_{L^{p(\cdot)}(X)} = \text{ínf}\{\lambda > 0 : \rho(f/\lambda) \leq 1\}.$$

These spaces, particularly when $X = \mathbb{R}^n$ and μ is Lebesgue measure, have been extensively studied for the past 20 years. It is well-known that if

$$p_+ = \operatorname*{ess\,sup}_{x\in X} p(x) < \infty,$$

then these spaces have many properties analogous to the classical Lebesgue spaces. In particular, the dual space $L^{p(\cdot)}(X)^*$ is isomorphic to $L^{p'(\cdot)}(X)$, where the exponent is defined pointwise by

$$\frac{1}{p(x)} + \frac{1}{p'(x)} = 1.$$

This is no longer true if $p^+ = \infty$, even if $p(x) < \infty$ everywhere, and it has been a long standing problem to characterize the dual space in this case. I will discuss recent progress on this problem, both for general measure spaces and for the special case of the discrete sequence spaces $\ell^{p(\cdot)}$, where $X = \mathbb{N}$ and μ is the discrete counting measure. We give a direct sum decomposition of the dual space as $L^{p'(\cdot)}(X) \oplus L^{p(\cdot)}_{germ}(X)$, where $L^{p(\cdot)}_{germ}(X)$, the germ space, intuitively consists of functions that "live" where the exponent function is unbounded. We give a number of properties of the germ space, particularly in the case of sequence spaces.

This talk is joint work with José Conde-Alonso, Jesús Ocariz, and Alex Amenta.