SYLLABUS PROPOSAL FOR THE ADVANCED COURSE IN ALGEBRA

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1. Geometric group theory

The aim of this course will be to serve as a brief introduction to geometric group theory, that is, to the study of infinite groups via their actions on metric spaces. The topic brings together ideas and techniques from several areas of mathematics including geometry, topology, dynamics and algebra.

We will begin by covering some of the more classical notions and problems from the area. We will then work through the fundamentals of groups acting on trees, establishing well-known results such as the Nielsen–Schreier theorem and Magnus solution to the word problem for one-relator groups. In the last part of the course, we will study groups from a coarse geometric perspective, introducing hyperbolic groups and developing some of their theory.

The topics we will cover are the following:

(1) Free groups.
(2) Group presentations.
(3) Dehns problems.
(4) Constructing new groups from old: amalgams and HNN-extensions.
(5) Group actions, free actions on trees, Nielsen–Schreier Theorem.
(6) The fundamental theorem of Bass–Serre theory.
(7) Magnus' solution to the word problem for one-relator groups.
(8) Cayley graphs.
(9) Quasi-isometries and the Švarc–Milnor Theorem.
(10) Hyperbolic groups.

Some previous background in topology, algebraic topology and group theory is preferable, but not essential. Some references for this course will be the following:

- Clara Löh. *Geometric group theory. An introduction.* Universitext. Springer, Cham, 2017. xi+389 pp. ISBN:978-3-319-72253-5.
- Martin R. Birdson, André Haefliger. *Metric spaces of non-positive curvature.* Grundlehren Math. Wiss., 319[Fundamental Principles of Mathematical Sciences] Springer-Verlag, Berlin, 1999. xxii+643 pp. ISBN:3-540-64324-9.
- Jean-Pierre Serre. *Trees.* Translated from the French original by John Stillwell. Springer Monogr. Math. Springer-Verlag, Berlin, 2003. x+142 pp. ISBN:3-540-44237-5.
- James McCool, Paul E. Schupp. *On one relator groups and HNN extensions.* J. Austral. Math. Soc.16(1973), 249–256.