Nonlinear diffusion and free boundary problems. A conference on the occasion of the 70th anniversary of Juan Luis Vázquez

UAM Madrid (Spain), May 17-19, 2017

- Xavier Cabré (ICREA-UPC, Barcelona, ES)
  Nonlocal minimal cones and surfaces with constant nonlocal mean curvature
- Luis Caffarelli (Univ. Texas at Austin, Austin TX, USA)
  Some non linear obstacle type problems
- José Antonio Carrillo (Imperial College, London, UK)
  Nonlinear aggregation-diffusions in the diffusion-dominated and fair-competitions regimes
- Panagiota Daskalopoulos (Columbia Univ., New York, USA)
  Fully non-linear geometric flows: regularity and asymptotic behavior
- Charlie Elliott (Univ. Warwick, Warwick, UK)
  PDEs on surfaces and evolving domains
- Alessio Figalli (ETH, Zurich, CH)
  The De Giorgi conjecture for the half-Laplacian in dimension 4
- Marek Fila (Comenius Univ., Bratislava, SK)
  Slow growth of solutions of super-fast diffusion equations with unbounded initial data
- Gabriele Grillo (Politecnico, Milano, IT)
  The porous medium equation on negatively curved manifolds
- Benoit Perthame (Univ. Pierre et Marie Curie, Paris, FR)
  Tumor growth: from compressible models to free boundaries
- Henrik Shahgholian (KTH, Stockholm, SE)
  Regularity of Free Boundaries (System case)
- Susanna Terracini (Univ. Torino, IT)
  On s-harmonic functions on cones
- Sylvia Serfaty (Courant Inst. NYU, New York, USA)
  Mean-Field Limits for Ginzburg-Landau vortices
- Luis Vega (BCAM, Bilbao, ES)
  Critical perturbations of Dirac Hamiltonians: selfadjointness and spectrum

Wednesday *May 17, 2017*	**Thursday** *May 18, 2017*	**Friday** *May 19, 2017*
10:30-11:00 **Opening**	10:00-10:50 **B. Perthame**	10:30-11:20 **J.A. Carrillo**
11:00-11:50 **X. Cabré**	10:50-11:20 **Coffee break**	11:20-11:50 **Coffee break**
11:50-12:20 **Coffee break**	11:20-12:10 **G. Grillo**	11:50-12:40 **C. Elliot**
12:20-13:10 **H. Shahgholian**	12:10-13:00 **S. Terracini**	12:40-13:30 **L. Caffarelli**
13:10-15:00 Lunch	13:00-15:00 Lunch	13:30-13:45 **Closing**
15:00-15:50 **S. Serfaty**	15:00-15:50 **P. Daskalopoulos**	/
15:50-16:40 **L. Vega**	15:50-16:20 Coffee break	/
/	16:20-17:10 **M. Fila**	/
/	17:10-18:00 **A. Figalli**	/
/	/	/
/	21:00 **Social Dinner**	/

Abstracts

- Xavier Cabré (ICREA-UPC, Barcelona, ES)
  Nonlocal minimal cones and surfaces with constant nonlocal mean curvature
  The talk will be concerned with hypersurfaces of $\mathbb{R}^n$ with zero, or constant, nonlocal mean curvature. This is the equation associated to critical points of the fractional s-perimeter. We prove that half spaces are the only stable s-minimal cones in $\mathbb{R}^3$ for s sufficiently close to 1. We will then turn to the nonlocal analogue of the Alexandrov result characterizing spheres as the only closed embedded hypersurfaces in $\mathbb{R}^n$ with constant mean curvature. Finally, we will describe results establishing the existence of periodic Delaunay-type cylinders in $\mathbb{R}^n$, as well as periodic lattices made of nearspheres, with constant nonlocal mean curvature.

- Luis Caffarelli (Univ. Texas at Austin, Austin TX, USA)
  Some non linear obstacle type problems
  We will discuss some recent work on problems of obstacle type, like two membranes or options interacting with each others or fully non linear discontinuities across a level surface

- José Antonio Carrillo (Imperial College, London, UK)
  Nonlinear aggregation-diffusions in the diffusion-dominated and fair-competitions regimes
  We analyse under which conditions equilibration between two competing effects, repulsion modelled by nonlinear diffusion and attraction modelled by nonlocal interaction, occurs. I will discuss several regimes that appear in aggregation diffusion problems with homogeneous kernels. I will first concentrate in the fair competition case distinguishing among porous medium like cases and fast diffusion like ones. I will discuss the main qualitative properties in terms of stationary states and minimizers of the free energies. In particular, all the porous medium cases are critical while the fast diffusion are not. In the second part, I will discuss the diffusion dominated case in which this balance leads to continuous compactly supported radially decreasing equilibrium configurations for all masses. All stationary states with suitable regularity are shown to be radially symmetric by means of continuous Steiner symmetrisation techniques. Calculus of variations tools allow us to show the existence of global minimizers among these equilibria. Finally, in the particular case of Newtonian interaction in two dimensions they lead to uniqueness of equilibria for any given mass up to translation and to the convergence of solutions of the associated nonlinear aggregation-diffusion equations towards this unique equilibrium profile up to translations as time tends to infinity. This talk is based on works in collaboration with S. Hittmeir, B. Volzone and Y. Yao and with V. Calvez and F. Hoffmann.

- Panagiota Daskalopoulos (Columbia Univ., New York, USA)
  Fully non-linear geometric flows: regularity and asymptotic behavior
  We will discuss the evolution of entire graphs by Inverse Mean Curvature flow and Gauss Curvature flow. We will address the questions of long time existence and regularity. We will also discuss the asymptotic behavior of solutions to the Gauss Curvature flow and the related problem of classification of solitons.

- Charlie Elliott (Univ. Warwick, Warwick, UK)
  PDEs on surfaces and evolving domains
  Many physical models give rise to the need to solve partial differential equations in time dependent regions. The complex morphology of biological membranes and cells coupled with biophysical mathematical models present significant computational challenges as evidenced within the Newton Institute programme "Coupling Geometric PDEs with Physics for Cell Morphology, Motility and Pattern Formation". In this talk we discuss the mathematical issues associated with the formulation of PDEs in time dependent domains in both flat and curved space. Here we are thinking of problems posed on time dependent d-dimensional hypersurfaces Gamma(t) in $\mathbb{R}^{d+1}$. The surface $\Gamma(t)$ may be the boundary of the bounded open bulk region $\Omega(t)$. In this setting we may also view $\Omega(t)$ as $(d+1)$-dimensional sub-manifold in $\mathbb{R}^{d+2}$. Using this observation we may develop a theory applicable to both surface and bulk equations. We will present an abstract framework for treating the theory of well- posedness of solutions to abstract parabolic partial differential equations on evolving Hilbert spaces using generalised Bochner spaces. This theory is applicable to variational formulations of PDEs on evolving spatial domains including moving hyper-surfaces. Our setting is abstract and not restricted to evolving domains or surfaces. Then we show well-posedness to a certain class of parabolic PDEs under some assumptions on the parabolic operator and the data. Specifically, we study in turn a surface heat equation, an equation posed on a bulk domain, a novel coupled bulk-surface system and an equation with a dynamic boundary condition. We give some background to applications in cell biology. We describe how the theory may be used in the development and numerical analysis of evolving surface finite element spaces which unifies the discretisation methodology for evolving surface and bulk equations. We give some computational examples from cell biology involving the coupling of surface evolution to processes on the surface.

- Alessio Figalli (ETH, Zurich, CH)
  The De Giorgi conjecture for the half-Laplacian in dimension 4
  The famous the Giorgi conjecture for the Allen-Cahn equation states that global monotone solutions are 1D if the dimension is less than 9. This conjecture is motivated by classical results about the structure of global minimal surfaces. The analogue of this conjecture in half-spaces can be reduced to study the problem in the whole space for the Allen-Cahn equation with the half-Laplacian. In this talk I will present a recent result with Joaquim Serra, where we prove the validity of the De Giorgi conjecture for stable solutions in dimension 3, that implies the result on monotone solutions in dimension 4.

- Marek Fila (Comenius Univ., Bratislava, SK)
  Slow growth of solutions of super-fast diffusion equations with unbounded initial data
  We study positive solutions of the super-fast diffusion equation in the whole space with initial data which are unbounded. We find an explicit dependence of the slow temporal growth rate of solutions on the initial spatial growth rate. A new class of self-similar solutions plays a significant role in our analysis. This is a joint work with Michael Winkler.

- Gabriele Grillo (Politecnico, Milano, IT)
  The porous medium equation on negatively curved manifolds
  We report on some recent results concerning existence, uniqueness and asymptotic behavior of solutions to the porous medium equation on classes of negatively curved manifolds. In particular we shall deal with the existence and uniqueness for (finite) measure data, upper and lower pointwise bounds on solutions, large data and blow-up in finite time.

- Benoit Perthame (Univ. Pierre et Marie Curie, Paris, FR)
  Tumor growth: from compressible models to free boundaries
  The growth of solid tumors can be described at a number of different scales from the cell to the organ. For a large number of cells, the 'fluid mechanical' approach has been advocated recently by many authors in mathematics or biophysics. Several levels of mathematical descriptions are commonly used, including possibly elasticity, visco-elastic laws, nutrients, active movement, surrounding tissue, and several other features.
  We will focuss on the links between two types of mathematical models. The 'microscopic' or 'compressible' description is at the cell population density level and a more macroscopic or 'incompressible' description is based on a free boundary problem close to the classical Hele-Shaw equation. In the stiff pressure limit, we are going to derive a weak formulation of the corresponding Hele-Shaw free boundary problem which describes possible singularities with uniqueness.
  Including additional features also opens other questions as circumstances in which singularities and instabilities may develop.

- Henrik Shahgholian (KTH, Stockholm, SE)
  Regularity of Free Boundaries (System case)
  I shall discuss the regularity theory for free boundary problems, in two particular cases:
  1) Obstacle type (joint with Andersson, Uraltseva, Weiss)
  2) Bernoulli type (Joint with Caffarelli, Yeressian)
  I shall mainly focus on the system case of these problems arising in minimization of functionals.

- Susanna Terracini (Univ. Torino, IT)
  On s-harmonic functions on cones
  We deal with functions satisfying

$$\begin{cases} (-\Delta)^s u_s = 0 & \text{in} \quad C, \\ u_s = 0 & \text{in} \quad \mathbb{R}^n \setminus C, \end{cases} \tag{1}$$

  where $s \in (0,1)$ and C is a given cone on $\mathbb{R}^n$ with vertex at zero. We are mainly concerned with the case when s approaches 1. These functions are involved in the study of the nodal regions in the case of optimal partitions and other free boundary problems and play a crucial role in the extension of the Alt-Caffarelli-Friedman monotonicity formula to the case of fractional diffusions. This is a joint work with Giorgio Tortone and Stefano Vita.

- Sylvia Serfaty (Courant Inst. NYU, New York, USA)
  Mean-Field Limits for Ginzburg-Landau vortices
  Ginzburg-Landau type equations are models for superconductivity, superfluidity, Bose-Einstein condensation. A crucial feature is the presence of quantized vortices, which are topological zeroes of the complex-valued solutions. This talk will review some results on the derivation of effective models to describe the statics and dynamics of these vortices, with particular attention to the situation where the number of vortices blows up with the parameters of the problem. In particular we will present new results on the derivation of mean field limits for the dynamics of many vortices starting from the parabolic Ginzburg-Landau equation or the Gross-Pitaevskii (=Schrodinger Ginzburg-Landau) equation.

- Luis Vega (BCAM, Bilbao, ES)
  Critical perturbations of Dirac Hamiltonians: selfadjointness and spectrum.
  I shall present some recent results about singular perturbations of Dirac operator and its connection with the boundedness of the Cauchy operator and Calderons projector operator. Also I will sketch the proof of an isoperimetric type inequality.